Guided Practice 3.8.2

Example 1

Find the constant ratio, write the explicit formula, and find the seventh term for the following geometric sequence.

3, 1.5, 0.75, 0.375, ...

1. Find the constant ratio by dividing two successive terms.

   $1.5 \div 3 = 0.5$

2. Confirm that the ratio is the same between all of the terms.

   $0.75 \div 1.5 = 0.5$ and $0.375 \div 0.75 = 0.5$

3. Identify the first term (a_1).

   $a_1 = 3$

4. Write the explicit formula.

   $a_n = a_1 \bullet r^{n-1}$ Explicit formula for any given geometric sequence

   $a_n = (3)(0.5)^{n-1}$ Substitute values for a_1 and n.

5. To find the seventh term, substitute 7 for n.

   $a_7 = (3)(0.5)^{7-1}$

   $a_7 = (3)(0.5)^6$ Simplify.

   $a_7 = 0.046875$ Multiply.

   The seventh term in the sequence is 0.046875.

Example 2

The fifth term of a geometric sequence is 1,792. The constant ratio is 4. Write an explicit formula for the sequence, and then write the corresponding exponential function.

1. The fifth term is 1,792; therefore $n = 5$ and $a_n = 1792$.

2. The constant ratio is 4; therefore, $r = 4$.

3. Substitute the known values into the explicit form of the formula and solve for a.

   $1792 = a(4)^{5-1}$ Substitute values.

   $1792 = 256a$ Simplify.

   $a = 7$

4. Write the explicit formula.

   $a_n = a_1 \bullet r^{n-1}$ Explicit formula for any given geometric sequence

   $a_n = 7(4)^{n-1}$ Substitute values.

5. Write the formula in function notation.

   $f(x) = 7(4)^{x-1}$

   Note that the domain of a geometric sequence is consecutive positive integers.

Example 3

A geometric sequence is defined recursively by $a_n = (a_{n-1})\left(-\frac{1}{3}\right)$, with $a_1 = 729$. Find the first five terms of the sequence, write an explicit formula to represent the sequence, and find the eighth term.

1. Using the recursive formula:

$$a_1 = 729$$
$$a_2 = (a_1)\left(-\frac{1}{3}\right)$$
$$a_2 = (729)\left(-\frac{1}{3}\right) = -243$$
$$a_3 = (-243)\left(-\frac{1}{3}\right) = 81$$
$$a_4 = (81)\left(-\frac{1}{3}\right) = -27$$
$$a_5 = (-27)\left(-\frac{1}{3}\right) = 9$$

The first five terms of the sequence are 729, –243, 81, –27, and 9.

2. The first term is $a_1 = 729$ and the constant ratio is $r = -\frac{1}{3}$, so the explicit formula is $a_n = (729)\left(-\frac{1}{3}\right)^{n-1}$.

3. Substitute 8 in for n and evaluate.

$$a_8 = (729)\left(-\frac{1}{3}\right)^{8-1}$$
$$a_8 = (729)\left(-\frac{1}{3}\right)^{7}$$
$$a_8 = -\frac{1}{3}$$

The eighth term of the sequence is $-\frac{1}{3}$.