Guided Practice 3.3.2

Example 1

Janie invests $1,300 at a rate of 2.6%, compounded monthly. The function that models this situation is $f(x)=1300\left(1+\frac{0.026}{12}\right)^{12x}$, where x represents time in years. What is the rate of change for the interval [1, 4]?

1. Determine the interval to be observed.

   The interval to be observed is [1, 4], or the interval where $1 \le x \le 4$.

2. Determine (x_1, y_1).

   The initial x-value is 1.

   Substitute the value 1 into the given function.

   $f(x)=1300\left(1+\frac{0.026}{12}\right)^{12x}$ Given function

   $f(1)=1300\left(1+\frac{0.026}{12}\right)^{12(1)}$ Substitute 1 for the value of x.

   $f(1)=1300\left(1+\frac{0.026}{12}\right)^{12}$ Simplify as needed.

   $f(1) = 1300(1.002)^{12}$

   $f(1) \approx 1334.21$

   (x_1, y_1) is (1, 1334.21).

3. Determine (x_2, y_2) by identifying the ending x-value of the interval and substituting it into the function.

   The ending x-value is 4.

   Substitute the value 4 into the given function.

$f(x)=1300\left(1+\dfrac{0.026}{12}\right)^{12x}$	Given function
$f(4)=1300\left(1+\dfrac{0.026}{12}\right)^{12(4)}$	Substitute 4 for the value of x.
$f(4)=1300\left(1+\dfrac{0.026}{12}\right)^{48}$	Simplify as needed.
$f(4)\approx 1442.32$	

   (x_2, y_2) is (4, 1442.32).

4. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula to calculate the rate of change.

$\dfrac{y_2-y_1}{x_2-x_1}$	Slope formula
$=\dfrac{1442.32-1334.21}{4-1}$	Substitute (1, 1334.21) and (4, 1442.32) for (x_1, y_1) and (x_2, y_2).
$=\dfrac{108.11}{3}$	Simplify as needed.
≈ 36.04	

   The rate of change for the interval [1, 4] is \$36.04 per year. ✓

Example 2

In 2008, about 66 million U.S. households had both landline phones and cell phones. This number decreased by an average of 5 million households per year. Use the table below to calculate the rate of change for the interval [2008, 2011].

Year (x)	Households in millions ($f(x)$)
2008	66
2009	61
2010	56
2011	51

1. Determine the interval to be observed.

   The interval to be observed is [2008, 2011], or where $2008 \le x \le 2011$.

2. Determine (x_1, y_1).

   The initial x-value is 2008 and the corresponding y-value is 66; therefore, (x_1, y_1) is (2008, 66).

3. Determine (x_2, y_2).

   The ending x-value is 2011 and the corresponding y-value is 51; therefore, (x_2, y_2) is (2011, 51).

4. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula to calculate the rate of change.

   $\dfrac{y_2 - y_1}{x_2 - x_1}$ Slope formula

   $= \dfrac{51-66}{2011-2008}$ Substitute (2008, 66) and (2011, 51) for (x_1, y_1) and (x_2, y_2).

   $= \dfrac{-15}{3}$ Simplify as needed.

   $= -5$

   The rate of change for the interval [2008, 2011] is 5 million households per year.

Example 3

A type of bacteria doubles every 36 hours. A Petri dish starts out with 12 of these bacteria. Use the table below to calculate the rate of change for the interval [2, 5].

Days (x)	Amount of bacteria ($f(x)$)
0	12
1	19
2	30
3	48
4	76
5	121
6	192

1. Determine the interval to be observed.

   The interval to be observed is [2, 5], or where $2 \le x \le 5$.

2. Determine (x_1, y_1).

   The initial x-value is 2 and the corresponding y-value is 30; therefore, (x_1, y_1) is (2, 30).

3. Determine (x_2, y_2).

   The ending x-value is 5 and the corresponding y-value is 121; therefore, (x_2, y_2) is (5, 121).

4. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula to calculate the rate of change.

   $\frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

   $= \frac{121-30}{5-2}$ Substitute (2, 30) and (5, 121) for (x_1, y_1) and (x_2, y_2).

   $= \frac{91}{3}$ Simplify as needed.

   ≈ 30.3

   The rate of change for the interval [2, 5] is approximately 30.3 bacteria per day.