## Guided Practice 3.2.1

**Example 1**

Complete the sequence by using recursion. What are the fifth, sixth, and tenth terms of the sequence?

$A = \{5, 9, 13, 17, a_5, a_6, 29, 33, 37, a_{10}\}$

1. First, determine the pattern.

   Is there a common difference or a common ratio? Subtract the second term from the first term and then continue that pattern to see if the difference between each pair of terms is the same.

   $a_2 - a_1 = 9 - 5 = 4$

   $a_3 - a_2 = 13 - 9 = 4$

   $a_4 - a_3 = 17 - 13 = 4$

   $a_8 - a_7 = 33 - 29 = 4$

   $a_9 - a_8 = 37 - 33 = 4$

   The common difference is 4.

2. Think about what it takes to get from one term to the next.

   The common difference is 4. This means that to get to the next term, add 4.

   $a_n = a_{n-1} + 4$

3. Use the recursive formula to calculate the fifth, sixth, and tenth terms in the sequence using the terms just before them.

   $a_5 = a_4 + 4 = 17 + 4 = 21$

   $a_6 = a_5 + 4 = 21 + 4 = 25$

   $a_{10} = a_9 + 4 = 37 + 4 = 41$

   The fifth, sixth, and tenth terms of the sequence are 21, 25, and 41.


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**Example 2**

Find the missing terms in the sequence using recursion.

$A = \{8, 13, 18, 23, a_5, a_6, a_7\}$

1. First look for the pattern. Is there a common difference or common ratio?

   $a_2 - a_1 = 5$

   $a_3 - a_2 = 5$

   $a_4 - a_3 = 5$

   The terms are separated by a common difference of 5.

   From this, we can deduce $a_n = a_{n-1} + 5$.

2. Use the formula to find the missing terms.

   $a_n = a_{n-1} + 5$

   $a_5 = a_4 + 5 = 23 + 5 = 28$

   $a_6 = a_5 + 5 = 28 + 5 = 33$

   $a_7 = a_6 + 5 = 33 + 5 = 38$

   The missing terms are 28, 33, and 38.


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**Example 3**

Find the missing terms in the sequence using recursion.

$B = \{6, 18, 54, 162, b_5, b_6, 4374, b_8\}$

1. First, determine the pattern.

   Is there a common difference or a common ratio?

   Subtract the second term from the first term and then continue that pattern to see if the difference between each pair of terms is the same.

   $b_2 - b_1 = 18 - 6 = 12$

   $b_3 - b_2 = 54 - 18 = 36$

   $b_4 - b_3 = 162 - 54 = 108$

   We can quickly see there is not a common difference; but, is there a common ratio?

   Divide each term by the term that precedes it to determine if there is a common ratio.

   $$\frac{b_2}{b_1} = \frac{18}{6} = 3$$

   $$\frac{b_3}{b_2} = \frac{54}{18} = 3$$

   $$\frac{b_4}{b_3} = \frac{162}{54} = 3$$

   The terms share a common ratio of 3.

2. Consider what function is performed to get from one term to the next.

   The common ratio is 3.

   The terms increase in value as the sequence progresses.

   To get to the next term, multiply by 3.

   $b_n = b_{n-1} \bullet 3$


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3. Use the recursive formula to find $b_5$, $b_6$, and $b_8$.

   $b_5 = b_4 \bullet 3 = 162 \bullet 3 = 486$

   $b_6 = b_5 \bullet 3 = 486 \bullet 3 = 1458$

   $b_8 = b_7 \bullet 3 = 4374 \bullet 3 = 13{,}122$

   The missing terms in the sequence are 486, 1,458, and 13,122.

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**Example 4**

Find the ninth term in the sequence given by $a_n = 3n + 1$. Then, graph the first 5 terms in the sequence.

1. Substitute 9 for $n$.

   $a_n = 3n + 1$

   $a_9 = 3(9) + 1$

   $a_9 = 27 + 1$

   $a_9 = 28$

2. Generate the first 5 terms of the sequence.

   $a_n = 3n + 1$

   $a_1 = 3(1) + 1 = 4$

   $a_2 = 3(2) + 1 = 7$

   $a_3 = 3(3) + 1 = 10$

   $a_4 = 3(4) + 1 = 13$

   $a_5 = 3(5) + 1 = 16$


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3. Create ordered pairs from the sequence.

   $n$ corresponds to $x$, and $a_n$ corresponds to $y$.

   $(n, a_n)$

   (1, 4)

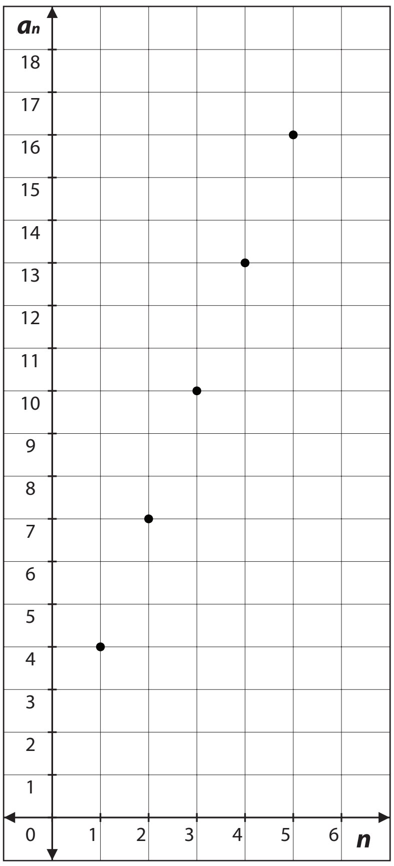
   (2, 7)

   (3, 10)

   (4, 13)

   (5, 16)

4. Plot the ordered pairs. Do not connect the points.



**Example 5**

Find the seventh term in the sequence given by $a_n = 3 \bullet 2^{n-1}$. Then, graph the first 5 terms in the sequence.

1. Substitute 7 for $n$.

   $a_n = 3 \bullet 2^{n-1}$

   $a_7 = 3 \bullet 2^{(7)-1} = 3 \bullet 2^6 = 3 \bullet 64 = 192$

2. Generate the first 5 terms of the sequence.

   $a_n = 3 \bullet 2^{n-1}$

   $a_1 = 3 \bullet 2^{(1)-1} = 3 \bullet 2^0 = 3 \bullet 1 = 3$

   $a_2 = 3 \bullet 2^{(2)-1} = 3 \bullet 2^1 = 3 \bullet 2 = 6$

   $a_3 = 3 \bullet 2^{(3)-1} = 3 \bullet 2^2 = 3 \bullet 4 = 12$

   $a_4 = 3 \bullet 2^{(4)-1} = 3 \bullet 2^3 = 3 \bullet 8 = 24$

   $a_5 = 3 \bullet 2^{(5)-1} = 3 \bullet 2^4 = 3 \bullet 16 = 48$

3. Create the ordered pairs from the sequence.

   $n$ corresponds to $x$, and $a_n$ corresponds to $y$.

   $(n, a_n)$

   (1, 3)

   (2, 6)

   (3, 12)

   (4, 24)

   (5, 48)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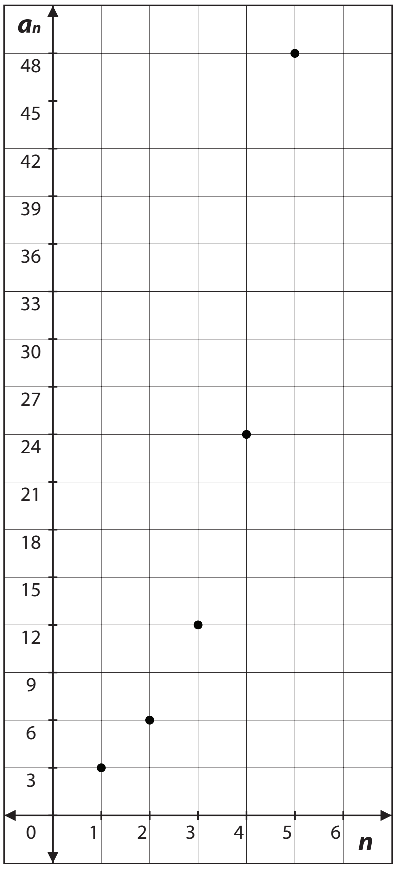
4. Plot the ordered pairs. Do not connect the points.





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