Guided Practice 3.6.1

Example 1

The starting balance of Anna's account is $1,250. She takes $30 out of her account each month. How much money is in her account after 1, 2, and 3 months? Find an equation to represent the balance in her account at any month.

1. Use the description of the account balance to find the balance after each month.

   Anna's account has $1,250. After 1 month, she takes out $30, so her account balance decreases by $30: $1250 – $30 = $1220.

   The new starting balance of Anna's account is $1,220. After 2 months, she takes out another $30. Subtract this $30 from the new balance of her account: $1220 – $30 = $1190.

   The new starting balance of Anna's account is $1,190. After 3 months, she takes out another $30. Subtract this $30 from the new balance of her account: $1190 – $30 = $1160.

2. Determine the independent and dependent quantities.

   The month number is the independent quantity, since the account balance depends on the month. The account balance is the dependent quantity.

3. Determine if there is a common difference or common ratio that describes the change in the dependent quantity.

   Organize your results in a table. Enter the independent quantity in the first column, and the dependent quantity in the second column. The balance at zero months is the starting balance of the account, before any money has been taken out. Because the independent quantity is changing by one unit, analyzing the differences between the dependent quantities will determine if there is a common difference between the dependent quantities.

Month	Account balance in dollars ($)	Difference
0	1250	
1	1220	1250 –1220 = –30
2	1190	1220 –1190 = –30
3	1160	1190 –1160 = –30

   The account balance has a common difference; it decreases by $30 for every 1 month. The relationship between the month and the account balance can be represented using a linear function.

4. Use the common difference to write an explicit equation.

   The general form of a linear function is: $f(x) = mx + b$, where m is the slope and b is the y-intercept. The common difference between the dependent terms in the pattern is the slope of the relationship between the independent and dependent quantities. Replace m with the slope, and replace x and $f(x)$ with an independent and dependent quantity pair in the relationship, such as (1, 1220). Solve for b.

   $1220 = (-30) \bullet (1) + b$

   $1250 = b$

   $f(x) = -30x + 1250$

5. Evaluate the equation to verify that it is correct.

   Organize your results in a table. Use the explicit equation to find each term. The terms that are calculated should match the terms in the original list.

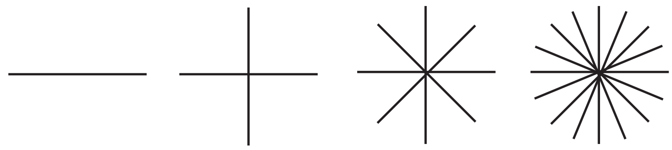
Month, x	**Account balance, $f(x)$, in dollars ($)**
0	$(-30) \bullet (0) + 1250 = 1250$
1	$(-30) \bullet (1) + 1250 = 1220$
2	$(-30) \bullet (2) + 1250 = 1190$
3	$(-30) \bullet (3) + 1250 = 1160$

   The pairs of dependent and independent quantities match the ones in the original pattern, so the explicit equation is correct.

   The balance in Anna's account can be represented using the equation $f(x) = -30x + 1250$. ✓

Example 2

Consider that the first figure below has two 180° angles, one on each side of the line segment. Each of these angles is then bisected or cut in half. This pattern continues, and the first 4 figures in the pattern are shown.



Write an equation to represent the relationship between the figure number and the number of angles in the figure.

1. Use the figures to determine the number of angles in figure numbers 1, 2, 3, and 4.

   Count the angles in each figure, taking into consideration the note that the first figure has 2 angles.

   Figure 1: 2 angles

   Figure 2: 4 angles

   Figure 3: 8 angles

   Figure 4: 16 angles

2. Define the independent and dependent quantity.

   The figure number is the independent quantity, and the number of angles in the figure is the dependent quantity.

3. Determine if there is a common difference or a common ratio that describes the change in the dependent quantity.

   Organize your results in a table. Enter the independent quantity in the first column, and the dependent quantity in the second column. The pattern appears to have a common ratio. Use a table to find the ratio between the terms. Divide the current term by the previous term.

Figure	Number of angles	Ratio
1	2	
2	4	$\frac{4}{2}=2$
3	8	$\frac{8}{4}=2$
4	16	$\frac{16}{8}=2$

   The common ratio between the dependent terms is 2.

4. Use the first pair of quantities and the common ratio to write an explicit equation.

   If b is the common ratio and a_1 is the value of the dependent quantity when the independent quantity is 1, then the general equation to represent the relationship is $f(x) = a_1b^{x-1}$. In this case, $a_1 = 2$ and $b = 2$, so the equation to represent the relationship is $f(x) = 2 \bullet 2^{x-1}$.

5. Evaluate the equation to verify that it is correct.

   Organize your results in a table. Use the explicit equation to find each dependent term. The terms that are calculated should match the terms in the original list.

Figure	Number of angles
1	$2 \bullet 2^{1-1} = 2$
2	$2 \bullet 2^{2-1} = 4$
3	$2 \bullet 2^{3-1} = 8$
4	$2 \bullet 2^{4-1} = 16$

   The dependent terms match the ones in the original pattern, so the explicit equation is correct.

   The relationship between the number of angles and the figure number can be described using the equation $f(x) = 2 \bullet 2^{x-1}$.

Example 3

A video arcade charges an entrance fee, then charges a fee per game played. The entrance fee is $5, and each game costs an additional $1. Find the total cost for playing 0, 1, 2, or 3 games. Describe the total cost with an explicit equation.

1. Use the description of the costs to find the total costs.

   If no games are played, then only the entrance fee is paid. The total cost for playing 0 games is $5.

   If 1 game is played, then the entrance fee is paid, plus the cost of one game. If each game is $1, the cost of one game is $1. The total cost is $5 + $1 = $6.

   If 2 games are played, then the entrance fee is paid, plus the cost of two games. If each game is $1, the cost of two games is $1 • 2 = $2. The total cost is $5 + $2 = $7.

   If 3 games are played, then the entrance fee is paid, plus the cost of three games. If each game is $1, the cost of three games is $1 • 3 = $3. The total cost is: $5 + $3 = $8.

2. Identify the independent and dependent quantities.

   The total cost is dependent on the number of games played, so the number of games is the independent quantity and the total cost is the dependent quantity.

3. Determine if there is a common difference or a common ratio between the dependent terms.

   There appears to be a common difference between the dependent terms. Use a table to find the difference between the dependent quantities. Subtract the current term from the previous term.

Games	Cost in dollars ($)	Difference
0	5	
1	6	$6 - 5 = 1$
2	7	$7 - 6 = 1$
3	8	$8 - 7 = 1$

   The common difference between the dependent terms is $1.

4. Use the common difference to write an explicit equation.

   The general form of a linear function is: $f(x) = mx + b$, where m is the slope and b is the y-intercept. The common difference between the dependent terms in the pattern is the slope of the relationship between the independent and dependent quantities. Replace m with the slope, and replace x and $f(x)$ with an independent and dependent quantity pair in the relationship, such as (1, 6). Solve for b.

   $6 = (1) \bullet (1) + b$

   $5 = b$

5. Evaluate the equation to verify that it is correct.

   Organize your results in a table. Use the explicit equation to find each term. The terms that are calculated should match the terms in the original list.

Games	Cost in dollars ($)
0	$1 \bullet (0) + 5 = 5$
1	$1 \bullet (1) + 5 = 6$
2	$1 \bullet (2) + 5 = 7$
3	$1 \bullet (3) + 5 = 8$

   The pairs of independent and dependent quantities match the ones in the original pattern, so the explicit equation is correct.

   The total cost of any number of games, x, can be represented using the equation: $f(x) = x + 5$.