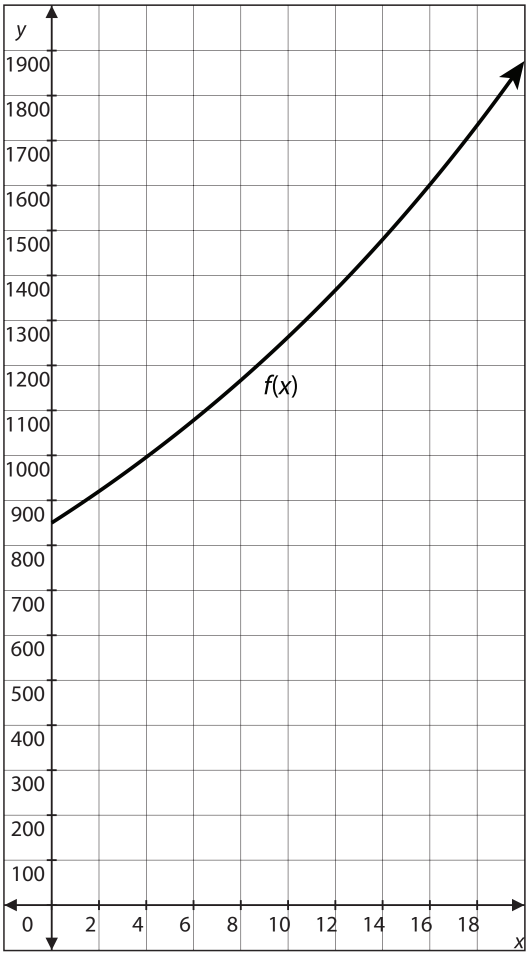
Guided Practice 3.5.2

Example 1

Compare the properties of each of the following two functions over the interval [0, 16].

Function A



Function B

x	$g(x)$
0	850
4	976.55
8	1121.94
12	1288.98
16	1480.88

1. Compare the y-intercepts of each function.

   Identify the y-intercept of the graphed function, $f(x)$.

   The graphed function appears to cross the y-axis at the point (0, 850).

   According to the table, $g(x)$ has a y-intercept of (0, 850).

   Both functions have a y-intercept of (0, 850).

2. Compare the rate of change for each function over the interval [0, 16].

   Calculate the rate of change over the interval [0, 16] for $f(x)$.

   Let $(x_1, y_1) = (0, 850)$.

   Determine (x_2, y_2) from the graph.

   The value of y when x is 16 is approximately 1,600.

   Let $(x_2, y_2) = (16, 1600)$.

   Calculate the rate of change using the slope formula.

$\frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$\frac{1600-850}{16-0}$	Substitute (0, 850) and (16, 1600) for (x_1, y_1) and (x_2, y_2).
$\frac{750}{16}$	Simplify as needed.
46.875	

   The rate of change for $f(x)$ is approximately 47.

   Calculate the rate of change over the interval [0, 16] for $g(x)$.

   Let $(x_1, y_1) = (0, 850)$.

   Determine (x_2, y_2) from the table.

   The value of y when x is 16 is 1,480.88.

   Let $(x_2, y_2) = (16, 1480.88)$.

(continued)

Calculate the rate of change using the slope formula.

$\dfrac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$\dfrac{1480.88 - 850}{16 - 0}$	Substitute (0, 850) and (16, 1480.88) for (x_1, y_1) and (x_2, y_2).
$\dfrac{630.88}{16}$	Simplify as needed.
39.43	

The rate of change for $g(x)$ is 39.43.

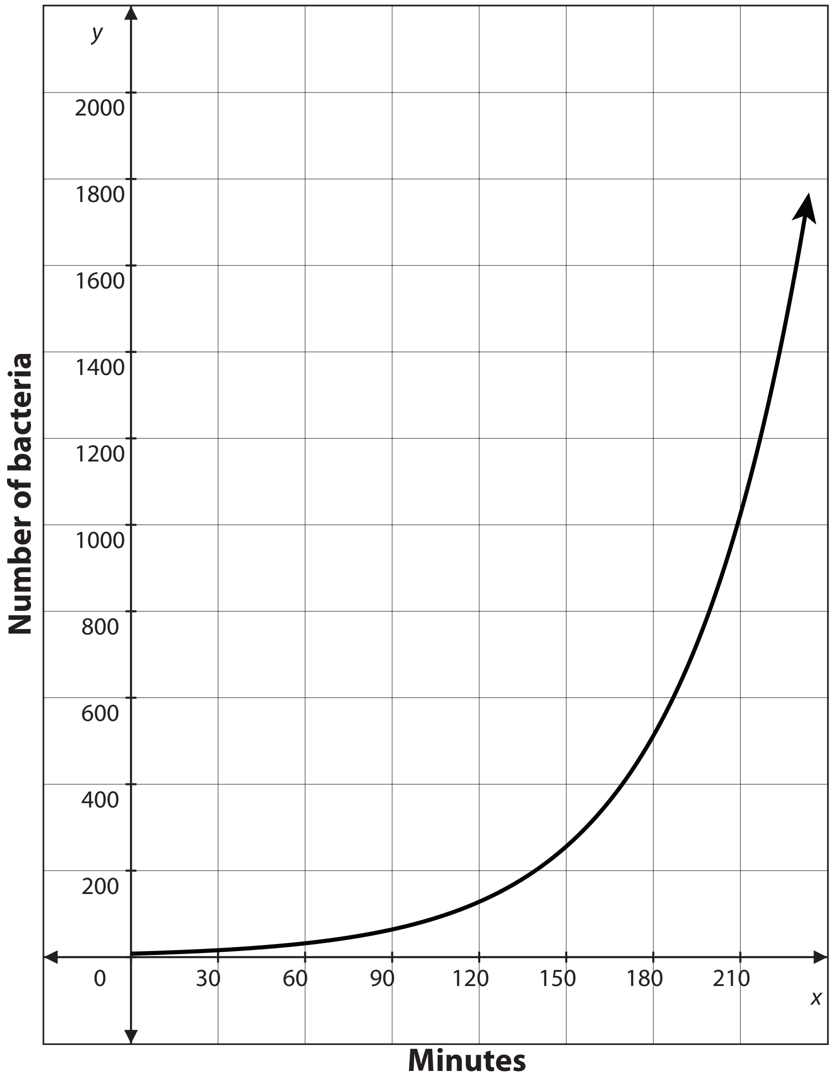
The rate of change for the graphed function, $f(x)$, is greater over the interval [0, 16] than the rate of change for the function in the table, $g(x)$.

3. Summarize your findings.

   The y-intercepts of both functions are the same; however, the graphed function, $f(x)$, has a greater rate of change over the interval [0, 16].

Example 2

A Petri dish contains 8 bacteria that double every 15 minutes. Compare the properties of the function that represents this situation to another population of bacteria, graphed below, that starts with 8 organisms over the interval [150, 210].



1. Compare the y-intercepts of each function.

   According to the scenario, the starting number of bacteria for both functions is 8; therefore, the y-intercept is (0, 8).

2. Compare the rate of change for each function over the interval [150, 210].

Calculate the rate of change over the interval [150, 210] for the graphed function.

Determine (x_1, y_1) from the graph.

The value of y when x is 150 is approximately 275.

Let $(x_1, y_1) = (150, 275)$.

Determine (x_2, y_2) from the graph.

The value of y when x is 210 is approximately 1,000.

Let $(x_2, y_2) = (210, 1000)$.

Calculate the rate of change using the slope formula.

$\dfrac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$\dfrac{1000-275}{210-150}$	Substitute (150, 275) and (210, 1000) for (x_1, y_1) and (x_2, y_2).
$\dfrac{725}{60}$	Simplify as needed.
≈ 12	

The rate of change for the graphed function is approximately 12 bacteria per minute.

To determine the rate of change for the function in the scenario, first write a function rule to represent the situation.

$$f(x) = 8(2)^{\frac{x}{15}}$$

Determine the value for y when x is 150 using the function.

$f(x) = 8(2)^{\frac{x}{15}}$	Original function
$f(x) = 8(2)^{\frac{150}{15}}$	Substitute 150 for x.
$f(150) = 8(2)^{10}$	Simplify as needed.

(*continued*)

$f(150) = 8(1024)$

$f(150) = 8192$

$(x_1, y_1) = (150, 8192)$

Determine the value for y when x is 210 using the function.

$f(x) = 8(2)^{\frac{x}{15}}$	Original function
$f(x) = 8(2)^{\frac{210}{15}}$	Substitute 210 for x.
$f(210) = 8(2)^{14}$	Simplify as needed.

$f(210) = 8(16{,}384)$

$f(210) = 131{,}072$

$(x_2, y_2) = (210, 131{,}072)$

Calculate the rate of change using the slope formula.

$\frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$\frac{131{,}072 - 8192}{210 - 150}$	Substitute (150, 8192) and (210, 131,072) for (x_1, y_1) and (x_2, y_2).
$\frac{122{,}880}{60}$	Simplify as needed.

2048

The rate of change for the function in the table is 2,048 bacteria per minute.

The rate of change for the graphed function is less steep over the interval [150, 210] than the rate of change for the scenario function.

3. Summarize your findings.

The y-intercepts of both functions are the same; however, the graphed function is less steep over the interval [150, 210]. The bacteria in the graphed function are doubling at a slower rate than the first function described.

Example 3

A pendulum swings to 90% of its previous height. Pendulum A starts at a height of 50 centimeters. Its height at each swing is modeled by the function $f(x) = 50(0.90)^x$. The height after every fifth swing of Pendulum B is recorded in the following table. Compare the properties of each function over the interval [5, 15].

x	$f(x)$
0	100
5	59.05
10	34.87
15	20.59
20	12.16

1. Compare the y-intercepts of each function.

   Identify the y-intercept of Pendulum A.

   The problem states that the pendulum starts at a height of 50 centimeters.

   The y-intercept of the function is (0, 50).

   Identify the y-intercept of Pendulum B.

   The value of $f(x)$ is 100 when x is 0.

   The y-intercept of the function is (0, 100).

2. Compare the rate of change for each function over the interval [5, 15].

   Calculate the rate of change over the interval [5, 15] for Pendulum A.

   Determine (x_1, y_1) from the function.

$f(x) = 50(0.90)^x$	Original function
$f(5) = 50(0.90)^5$	Substitute 5 for x.
$f(5) = 29.52$	Simplify as needed.

   Let $(x_1, y_1) = (5, 29.52)$.

   Determine (x_2, y_2) from the function.

$f(x) = 50(0.90)^x$	Original function
$f(15) = 50(0.90)^{15}$	Substitute 15 for x.
$f(15) \approx 10.29$	Simplify as needed.

   The value of y when x is 15 is approximately 10.29.

   Let $(x_2, y_2) = (15, 10.29)$.

   Calculate the rate of change using the slope formula.

$\dfrac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$\dfrac{10.29 - 29.52}{15 - 5}$	Substitute (5, 29.52) and (15, 10.29) for (x_1, y_1) and (x_2, y_2).
$\dfrac{-19.23}{10} = -1.923$	Simplify as needed.

   The rate of change for Pendulum A's function is approximately –1.923 centimeters per swing.

   (*continued*)

Calculate the rate of change over the interval [5, 15] for Pendulum B.

Let $(x_1, y_1) = (5, 59.05)$.

Let $(x_2, y_2) = (15, 20.59)$.

Calculate the rate of change using the slope formula.

$\dfrac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$\dfrac{20.59 - 59.05}{15 - 5}$	Substitute (5, 59.05) and (15, 20.59) for (x_1, y_1) and (x_2, y_2).
$\dfrac{-38.46}{10} = -3.846$	Simplify as needed.

The rate of change for Pendulum B's function is approximately –3.846 centimeters per swing.

The rate of change for Pendulum B is greater over the interval [5, 15] than the rate of change for Pendulum A.

3. Summarize your findings.

   The y-intercept of Pendulum A is less than the y-intercept of Pendulum B. This means that Pendulum B begins higher than Pendulum A. The rate of change for Pendulum A is less than the rate of change for Pendulum B. This means that Pendulum B is losing height faster than Pendulum A.