Guided Practice 2.3.2

Example 1

Solve the following system of inequalities graphically:

$$\begin{cases} x + y > 10 \\ 2x - 4y > 5 \end{cases}$$

1. Graph the line $x + y = 10$. Use a dashed line because the inequality is non-inclusive (greater than).

2. Shade the solution set. First pick a test point. Choose a point that is on either side of the line.

   Test point: (0, 0)

3. Then, substitute that point into the inequality $x + y > 10$. If the test point makes the inequality true, shade the region that contains that point. If the test point makes the inequality false, shade on the opposite side of the line.

   $x + y > 10$

   $(0) + (0) \stackrel{?}{>} 10$

   $0 \not> 10$

4. Since the point (0, 0) makes the inequality false, shade the opposite side of the line. The shaded region represents the solutions for $x + y > 10$.

5. Graph the line $2x - 4y = 5$ on the same coordinate plane. Use a dashed line because the inequality is non-inclusive (greater than).

6. Shade the solution set. First pick a test point. Choose a point that is on either side of the line.

   Test point: (0, 0)

7. Then, substitute that point into the inequality $2x - 4y > 5$. If the test point makes the inequality true, shade the region that contains that point. If the test point makes the inequality false, shade on the opposite side of the line.

   $2x - 4y > 5$

   $2(0) - 4(0) > 5$

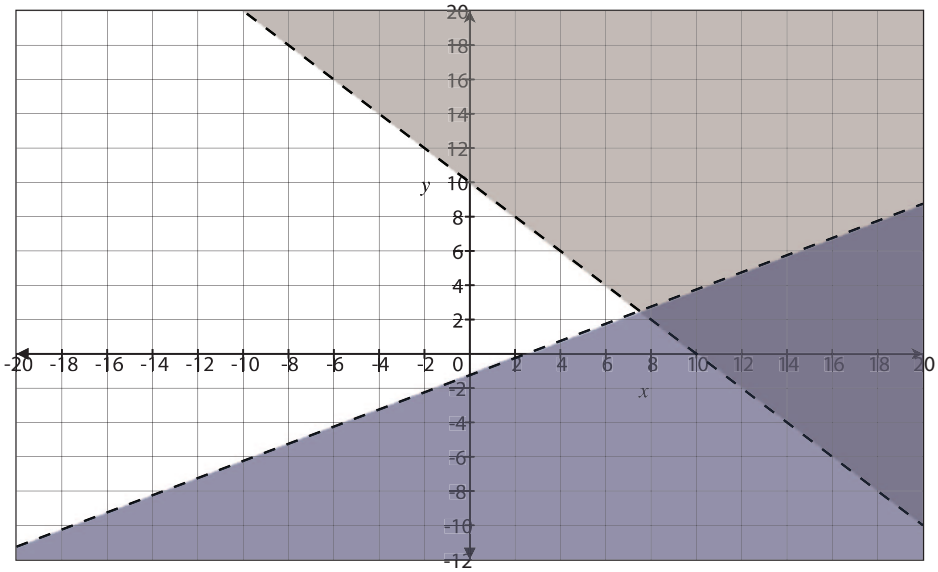
   $0 \not> 5$

8. Since the point (0, 0) makes the inequality false, shade the opposite side of the line. The second shaded region represents the solutions for $2x - 4y > 5$.

9. Find the solutions to the system. The overlap of the two shaded regions, which is darker, represents the solutions to the system:

   $\begin{cases} x + y > 10 \\ 2x - 4y > 5 \end{cases}$.

   A possible solution to this system is (14, 2) because it satisfies both inequalities.



Example 2

Solve the following system of inequalities graphically:

$$\begin{cases} -4x + 2y \geq 20 \\ x - y \geq 10 \end{cases}$$

1. Graph the line $-4x + 2y = 20$. Use a solid line because the inequality is inclusive (greater than *or* equal to).

2. Shade the solution set. First, pick a test point. Choose a point that is on either side of the line.

   Test point: (0, 0)

3. Then, substitute that point into the inequality $-4x + 2y \geq 20$. If the test point makes the inequality true, shade the region that contains that point. If the test point makes the inequality false, shade on the opposite side of the line.

   $-4x + 2y \geq 20$

   $-4(0) + 2(0) \geq 20$

   $0 \not\geq 20$

4. Since the point (0, 0) makes the inequality false, shade the opposite side of the line. The second shaded region represents the solutions for $-4x + 2y \geq 20$.

5. Graph the line $x - y = 10$. Use a solid line because the inequality is inclusive (greater than *or* equal to).

6. Shade the solution set. First, pick a test point. Choose a point that is on either side of the line.

   Test point: (0, 0)

Instruction

7. Then, substitute that point into the inequality $x - y \geq 10$. If the test point makes the inequality true, shade the region that contains that point. If the test point makes the inequality false, shade on the opposite side of the line.

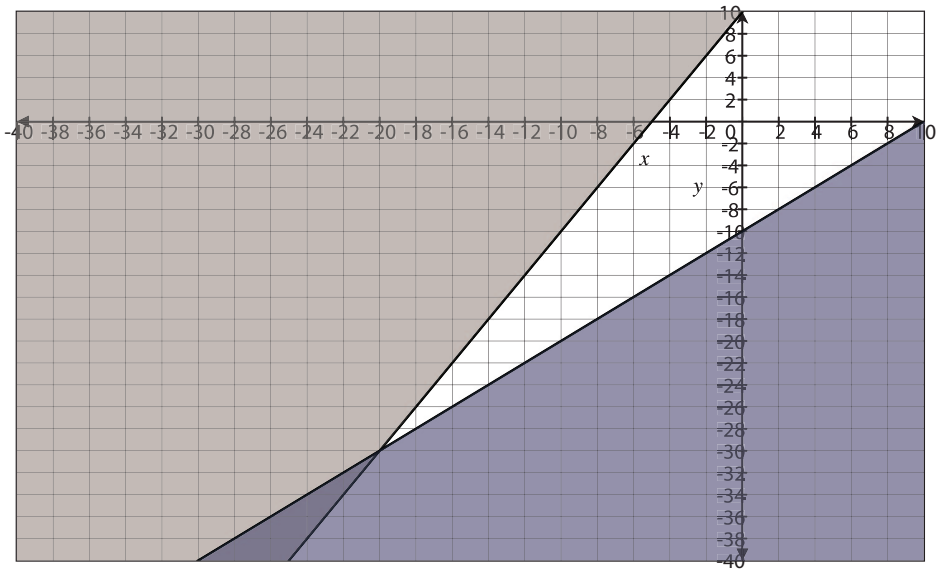
   $x - y \geq 10$

   $(0) - (0) \geq 10$

   $0 \not\geq 10$

8. Since the point (0, 0) makes the inequality false, shade the opposite side of the line. The second shaded region represents the solutions for $x - y \geq 10$.

9. Find the intersection of the two shaded regions. The overlap of the first two shaded regions, which is darker, represents the solutions to the system $\begin{cases} -4x + 2y \geq 20 \\ x - y \geq 10 \end{cases}$. A possible solution is (–24, –36).



Instruction

Example 3

Solve the following system of inequalities graphically:

$$\begin{cases} y \le x + 2 \\ y \ge 2x - 10 \\ x \le 0 \\ y \ge 0 \end{cases}$$

1. Graph the line $y = x + 2$. Use a solid line because the inequality is inclusive (less than *or* equal to).

2. Shade the solution set. First, pick a test point. Choose a point that is on either side of the line.

   Test point: (0, 0)

3. Then, substitute that point into the inequality $y \le x + 2$. If the test point makes the inequality true, shade the region that contains that point. If the test point makes the inequality false, shade on the opposite side of the line.

   $y \le x + 2$

   $0 \le 0 + 2$

   $0 \le 2$

4. Since the point (0, 0) makes the inequality true, shade the region that contains the point. The shaded region represents $y - x + 2$.

5. Graph the line $y \ge 2x - 10$. Use a solid line because the inequality is inclusive (greater than *or* equal to).

6. Shade the solution set. First, pick a test point. Choose a point that is on either side of the line.

   Test point: (0, 0)

Instruction

7. Then, substitute that point into the inequality $y \geq 2x - 10$. If the test point makes the inequality true, shade the region that contains that point. If the test point makes the inequality false, shade on the opposite side of the line.

   $y \geq 2x - 10$

   $(0) \geq 2(0) - 10$

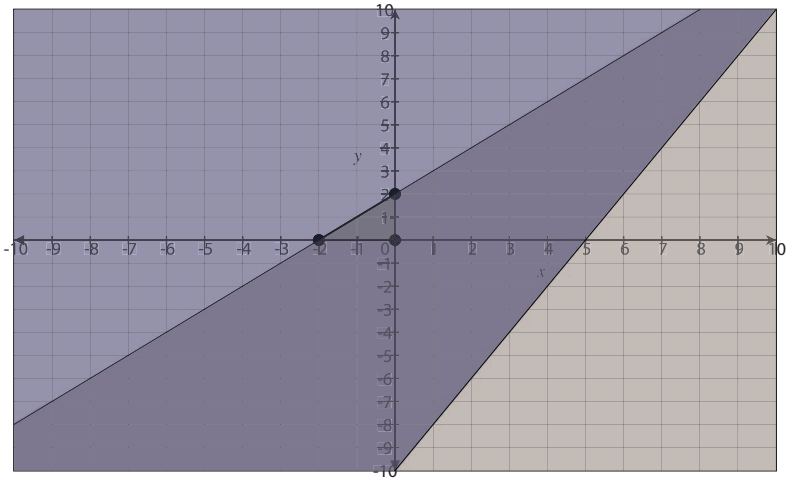
   $0 \geq -10$

8. Since the point (0, 0) makes the inequality true, shade the region that contains the point.

9. Find the intersection of the two shaded regions. The dark gray region represents where solutions to $y \leq x + 2$ and $y \geq 2x - 10$ overlap.

10. You also have the constraints of $x \leq 0$ and $y \geq 0$. This decreases the size of the dark gray region that will satisfy all inequalities in the system.

$$\begin{cases} y \leq x + 2 \\ y \geq 2x - 10 \\ x \leq 0 \\ y \geq 0 \end{cases}$$



Therefore, the solutions to the system are within the dark triangular region bounded by points (–2, 0), (0, 0), and (0, 2). A possible solution to this system is (–0.5, 0.5).

Instruction

Example 4

An artist wants to analyze the time that he spends creating his art. He makes oil paintings and watercolor paintings. The artist takes 8 hours to paint an oil painting. He takes 6 hours to paint a watercolor painting. He has set aside a maximum of 24 hours per week to paint his paintings. The artist then takes 2 hours to frame and put the final touches on his oil paintings. He takes 3 hours to frame and put the final touches on his watercolor paintings. He has set aside a maximum of 12 hours per week for framing and final touch-ups. Write a system of inequalities that represents the time the artist has to complete his paintings. Graph the solution.

1. Create the system of inequalities.

   Let x = the number of oil paintings he makes.

   Let y = the number of watercolor paintings he makes.

   It might be helpful to create a table:

	Oil (x)	Watercolor (y)	Total
Paint	8	6	24
Frame	2	3	12

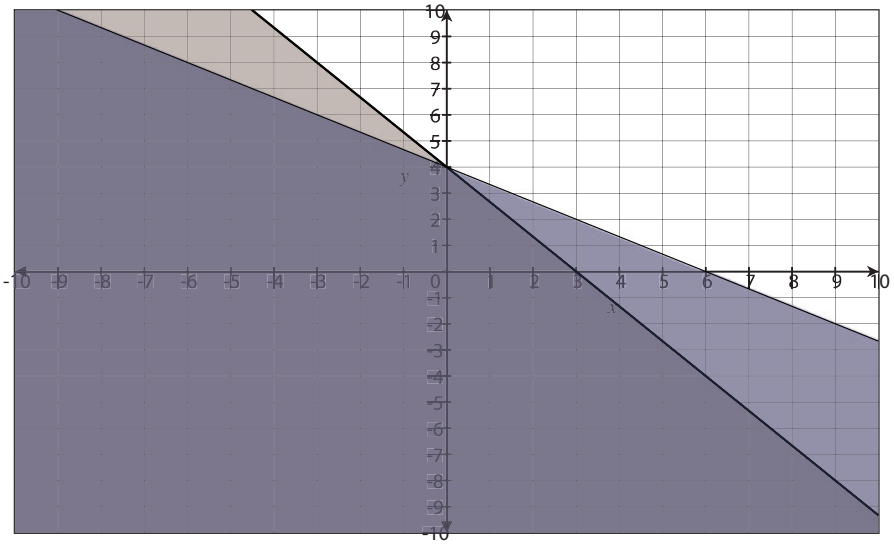
$$\begin{cases} 8x+6y \le 24 \\ 2x+3y \le 12 \end{cases}$$

   Now, think about what must always be true of creating the paintings: there will never be negative paintings. Add these two constraints to the system: $x \ge 0$ and $y \ge 0$.

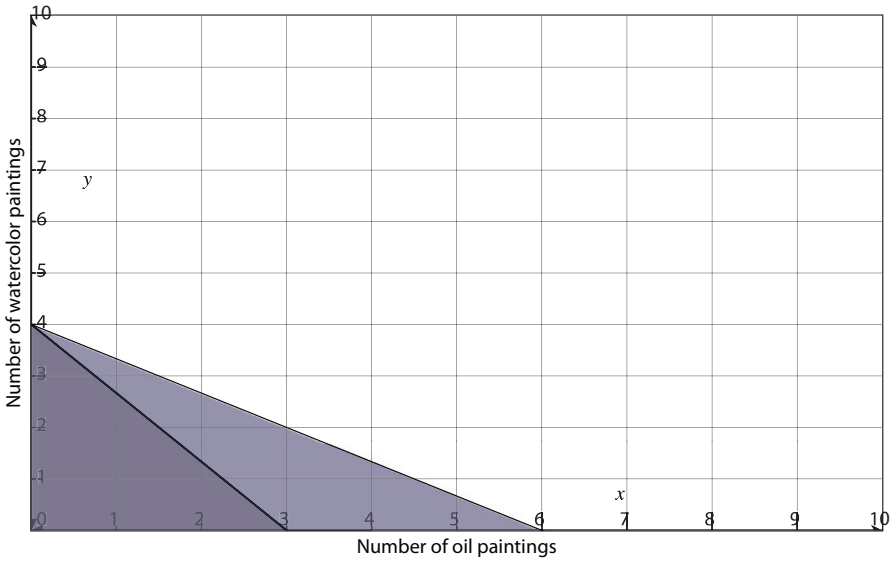
$$\begin{cases} 8x+6y \le 24 \\ 2x+3y \le 12 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

2. Graph the system on the same coordinate plane.

   Start by graphing the first two inequalities.



   Now apply the last two constraints: $x \geq 0$ and $y \geq 0$. This means the solution lies in the first quadrant.



   The solution is the darker shaded region; any points that lie within it are solutions to the system. The point (1, 1) is a solution because it satisfies both inequalities. The artist can create 1 oil painting and 1 watercolor painting given the time constraints he has. Or, he can create no oil paintings and 4 watercolor paintings, (0, 4). However, he cannot create 4 oil paintings and 1 watercolor painting, because the point (4, 1) only satisfies one inequality and does not lie in the darker shaded region.