Guided Practice 2.1.2

Example 1

Solve the equation $5x + 9 = 2x - 36$.

1. Move the variable to one side of the equation.

   Notice that the same variable, x, is on both sides of the equation: $5x$ is on the left of the equation and $2x$ is on the right. It makes no difference whether you choose to have the variables on the left or on the right; your solution will remain the same. It's common to have the variable on the left, but not necessary.

   It's often easier to move the variable with the smallest coefficient to the opposite side of the equation. Here, $2x$ is smaller than $5x$, so let's move $2x$.

   $2x$ is positive, so to get it to the other side of the equal sign you will need to subtract it from both expressions in the equation.

   It helps to line up what you are subtracting with the terms that are similar in order to stay organized. In this case, we are subtracting variables from variables.

$$\begin{array}{rcl} 5x+9 & = & 2x-36 \\ \underline{-2x} & & \underline{-2x} \\ 3x+9 & = & -36 \end{array}$$

   When $2x$ is subtracted, it's important not to forget the remaining terms of each expression. Look out for subtraction signs that now act as negative signs. Here, since 36 was originally being subtracted from $2x$, the subtraction sign left behind makes 36 negative.

   $3x + 9 = -36$

2. Continue to solve the equation $3x + 9 = -36$.

   To isolate x, subtract 9 from both expressions in the equation.

   $$\begin{array}{rcl} 3x+9 & = & -36 \\ \underline{\quad -9} & & \underline{-9} \\ 3x & = & -45 \end{array}$$

   Divide both expressions by the coefficient of x, 3.

   $$\frac{3x}{3}=\frac{-45}{3}$$

   $$x=-15$$

3. The solution to the equation $5x + 9 = 2x - 36$ is $x = -15$.

   A quick check will verify this. Substitute –15 for all instances of x in the original equation, and then evaluate each expression.

   $5x + 9 = 2x - 36$

   $5(-15) + 9 = 2(-15) - 36$

   $-75 + 9 = -30 - 36$

   $-66 = -66$

   Our check verified that both sides of the equation are still equal; therefore, $x = -15$ is correct.

Example 2

Solve the equation $7x + 4 = -9x$.

1. Move the variable to one side of the equation.

   Notice that the variable x is on both sides of the equation. Again, it makes no difference mathematically which term, $7x$ or $-9x$, you choose to eliminate.

   $-9x$ is the only term in the expression on the right side of the equation, so it may be easier to eliminate $7x$ from the left side.

   Subtract $7x$ from both expressions of the equation.

   $$\begin{array}{r} 7x+4=-9x \\ \underline{-7x\qquad -7x} \\ 4=-16x \end{array}$$

2. Continue to solve the equation $4 = -16x$.

   To isolate x, divide both expressions by the coefficient of x, -16.

   $$\frac{4}{-16}=\frac{-16x}{-16}$$

   $$-\frac{1}{4}=x$$

   $$x=-\frac{1}{4}$$

3. The solution to the equation $7x + 4 = -9x$ is $x = -\frac{1}{4}$.

   A quick check will verify this. Substitute $-\frac{1}{4}$ for all instances of x in the original equation, and then evaluate each expression.

$7x + 4 = -9x$	Original equation
$7\left(-\frac{1}{4}\right) + 4 = -9\left(-\frac{1}{4}\right)$	Substitute $-\frac{1}{4}$ for x.
$\left(-\frac{7}{4}\right) + 4 = \frac{9}{4}$	Multiply.
$\left(-\frac{7}{4}\right) + \left(\frac{16}{4}\right) = \frac{9}{4}$	Convert the whole number 4 to a fraction with a common denominator.
$\frac{9}{4} = \frac{9}{4}$	Add and review the result.

   Our check verified that both sides of the equation are still equal; therefore, $x = -\frac{1}{4}$ is correct. ✓

Example 3

Solve the equation $2(3x + 1) = 6x + 14$.

1. Simplify each side of the equation.

   Notice that the variable x is on both sides of the equation. Also notice the set of parentheses in the expression on the left side of the equation.

   Eliminate the parentheses by first distributing the 2 over $3x + 1$.

$2(3x + 1) = 6x + 14$	Multiply 2 and $3x + 1$.
$6x + 2 = 6x + 14$	

2. Move the variable to one side of the equation. Again, you need to eliminate one of the terms with the variable x. Subtract $6x$ from both expressions of the equation.

$$\begin{array}{r} 6x+2=6x+14 \\ \underline{-6x \qquad -6x \qquad} \\ 2=14 \end{array}$$

3. This equation has no solution.

   Subtracting the variables gives us an impossible result: 2 = 14. This is not a true statement; therefore, there are no solutions to this equation.

Example 4

Solve the equation $3(4x + 2) = 12x + 6$.

1. Simplify each side of the equation.

   Notice that the variable x is on both sides of the equation. Also notice the set of parentheses in the expression on the left side of the equation.

   Eliminate the parentheses by first distributing the 3 over $4x + 2$.

   $3(4x + 2) = 12x + 6$ Multiply 3 and $4x + 2$.

   $12x + 6 = 12x + 6$

2. This equation has an infinite number of solutions.

   The expressions on either side of the equation are the same. This means that any value substituted for x will result in a true statement.

Example 5

Solve the literal equation $A=\frac{1}{2}(b_1+b_2)h$ for b_1.

1. Isolate b_1.

   As we saw in Unit 1, to solve literal equations for a specific variable, we follow the same steps as solving equations.

   In this equation, we could distribute $\frac{1}{2}$ over $b_1 + b_2$, but this may cause more work for us. Instead, let's get rid of the fraction. Multiply both sides of the equation by the inverse of $\frac{1}{2}$, or 2.

   $$A=\frac{1}{2}(b_1+b_2)h$$

   $$2\bullet A=2\bullet\frac{1}{2}(b_1+b_2)h$$

   $$2A=(b_1+b_2)h$$

2. Again, you could distribute h over $b_1 + b_2$, but it's more efficient to divide both sides of the equation by h.

   $$2A=(b_1+b_2)h$$

   $$\frac{2A}{h}=\frac{(b_1+b_2)h}{h}$$

   $$\frac{2A}{h}=b_1+b_2$$

3. Finally, to solve the equation for b_1, subtract b_2 from both expressions of the equation.

$$\frac{2A}{h} = b_1 + b_2$$
$$\underline{-b_2 \qquad -b_2}$$
$$\frac{2A}{h} - b_2 = b_1$$

4. The equation $A = \frac{1}{2}(b_1 + b_2)h$ solved for b_1 is $\frac{2A}{h} - b_2 = b_1$.

   The equation can be rewritten as $b_1 = \frac{2A}{h} - b_2$ using the symmetric property of equality. ✓