

## Relationships Between Quantities

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Reason quantitatively and use units to solve problems.**

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling. ★

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★

**Interpret the structure of expressions**

**MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context. ★ *(Emphasis on linear expressions and exponential expressions with integer exponents.)*

**MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients. ★ *(Emphasis on linear expressions and exponential expressions with integer exponents.)*

**MCC9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity. ★ *(Emphasis on linear expressions and exponential expressions with integer exponents.)*

**Create equations that describe numbers or relationships**

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions. ★

**MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★ *(Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.)*

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. ★ *(Limit to linear equations and inequalities.)*

**MCC9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★ *(Limit to formulas with a linear focus.)*

**Lesson 1.0 Basics of Algebra**

**Algebra** is the branch of mathematics that deals with relationships between numbers, utilizing letters and other symbols to represent specific sets of numbers, or to describe a pattern of relationship between numbers.

A **variable** is a letter or symbol used to represent a number.

An **algebraic expression** is a collection of numbers, variables, and operations.

Examples: 1.  $a + b$       2.  $5 - x$       3.  $2y - 3$       4.  $x^3 - y^2z^5 - 8$       5.  $5(x - 4)$

Terms are parts in an expression separated by + or – signs.

**In the first 5 examples above, how many terms does each have?**

1.                      2.                      3.                      4.                      5.

Factors are two or more numbers or variables multiplied. **How many factors are in each of the following expressions?**

1.  $2y$                       2.  $4xyz$                       3.  $5(x + 6)$

A coefficient is a number multiplied by a variable.

**What is the coefficient of x?**

1.  $x$                       2.  $\frac{1}{2}x$                       3.  $3 + 5x$                       4.  $(1 + 5)x$

## Lesson 1.1 Understanding Dependency and Functions

**What is dependency?** In order to cook a meal you have to go to the grocery store first. That means cooking the meal depends on going to the grocery store. Thus cooking the meal is the dependent variable and going to the grocery store is the independent variable. One task depends on another task. In the following exercise you are asked to find the dependent and independent variable. Underline the dependent variable and circle the independent variable as shown in No.1.

1. Number of hours worked and money earned.
2. The velocity of a car and its horsepower.
3. The number of people working on a particular job and the time it takes to complete a job.
4. The number of games played and the number of points scored.
5. The speed of a car and how far the driver pushes down on the gas pedal.

Now come up with 3 scenarios yourself and highlight the dependent and independent variable.

6.

7.

8.

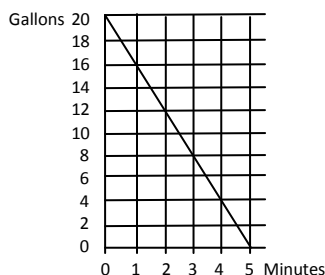
### Example

You drain a 20-gallon aquarium at a constant rate of 4 gallons per minute.

- a. What is the independent quantity?
- b. What is the dependent quantity?
- c. Draw a graph showing the independent quantity on the x-axis and dependent quantity on the y-axis.
- d. What is the slope? Include the units in your answer.

### Solution:

- a. independent quantity: time (minutes) – because time is independent of any event
- b. dependent quantity: water (gallons) – because the amount of water left depends on the time passed
- c.



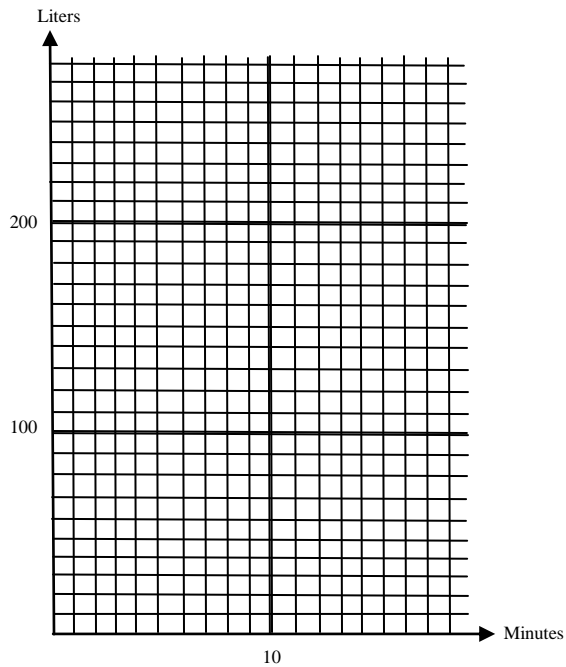
- d. The slope is -4 gallons per minute or  $-4 \frac{\text{gallons}}{\text{minute}}$ . That means, for every 4 gallons you lose (you move down by 4 units), you run 1 minute to the right.

9. You drain a 200-liter aquarium at a constant rate of 20 liters per minute.

a. What is the independent quantity?

b. What is the dependent quantity?

c. Draw a graph showing the independent quantity on the x-axis and dependent quantity on the y-axis.



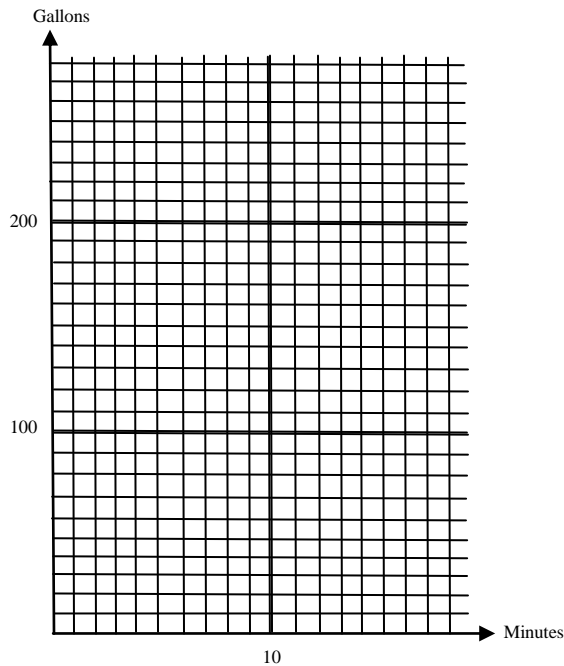
d. What is the slope? Include the units in your answer.

e. By using the graph, how much water is left in the aquarium after 3 minutes?

f. By using the graph, how long does it take that only 80 liters are left in the aquarium?

10. You **fill** (positive slope) a 180-gallon water tank at a constant rate of 30 gallons per minute.

- What is the independent quantity?
- What is the dependent quantity?
- Draw a graph showing the independent quantity on the x-axis and dependent quantity on the y-axis.



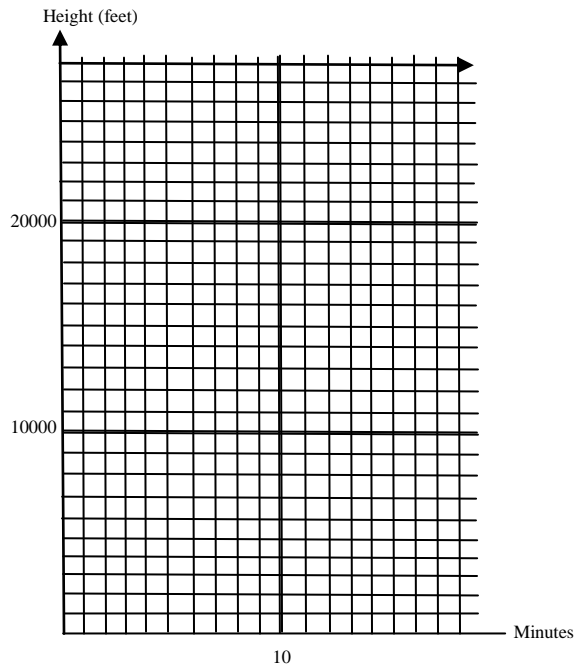
d. What is the slope? Include the units in your answer.

e. Use the graph to find out the following:

- The time it takes to completely fill the tank: \_\_\_\_\_
- The time it takes to fill the tank to 120 gallons: \_\_\_\_\_
- The amount of gallons in the tank after 5 minutes: \_\_\_\_\_
- The time it takes to fill the tank to 100 gallons: \_\_\_\_\_

**11.** From a height of 20,000 feet a plane descends at a rate of 2500 feet per minute until it lands.

- What is the independent quantity?
- What is the dependent quantity?
- Draw a graph showing the independent quantity on the x-axis and dependent quantity on the y-axis.



d. What is the slope? Include the units in your answer.

e. Determine how long it takes for the plane to descend to 15,000 feet.

f. Determine how long it takes for the plane to descend to 7,000 feet.

h. How high is the plane after 3 minutes of descend?

## Lesson 1.2 Use Formulas and Units to Solve Problems

<b>Example</b>	Use the Formula of Ohm's Law to solve for current, volts, and resistance.	
	An electrical circuit at a current of 1.5 Ampere and a voltage of 10 Volts. What is the resistance in the circuit?	
<b>Solution</b>	Ohm's law is given with the equation	
	$I = \frac{V}{R}$ where $I = \text{current(Amps)}$ , $V = \text{voltage(Volts)}$ , $R = \text{resistance(Ohm)}$	
Step 1	Write down formula	$I = \frac{V}{R}$
Step 2	Solve for the variable required	$R \times I = \frac{V}{R} \times R$ by first multiplying both sides by $R$
Step 3	Rewrite	$R \times I = V$
Step 4	Divide by $I$	$\frac{R \times I}{I} = \frac{V}{I}$
Step 5	Rewrite	$R = \frac{V}{I}$
Step 6	Plug in values and solve	$R = \frac{V}{I} = \frac{10}{1.5} \approx 6.67\Omega$ ( $\Omega = \text{Ohm}$ )

### PROBLEMS

1. An electrical circuit has 6 Volt and a current of 2 Amps. What is the resistance of the circuit?
2. An electrical circuit has 3 Volt and a resistance of 9 Ohm. What is the current of the circuit?
3. A businesswoman is reimbursed \$50 a day for food and lodging. She also receives \$0.55 for each mile driven. She drives 384 miles and is reimbursed \$361.20. How many days was the trip?
4. You want to buy a TV for \$990. You have saved \$620. You plan to save \$10 per week. How many weeks will it take to save for the TV?

5. It takes 30 minutes to drive 27 miles. What is your average speed in miles/hour? (Remember: 30 minutes =  $\frac{1}{2}$  hour)

$$\text{DISTANCE} = \text{SPEED} \times \text{TIME}$$

6. Your cell phone plan gives you 500 free minutes a month for \$30 and charges \$0.20 for every minute over 500 minutes. How long can you talk on the phone each month if you want to spend \$45 on your phone bill?

### Example

Isaac Newton derived a famous formula which is known today as Newton's 2<sup>nd</sup> law:

$$F = ma \quad \text{where } F = \text{force (N)}, m = \text{mass (kg)}, \text{ and } a = \text{acceleration (m/s}^2\text{)}.$$

That means the acceleration of an object (with mass,  $m$ ) is dependent of the force upon that object. In other words the acceleration of an object will be higher if you apply a larger force.

1. What are the units of Newton (N) in terms of kg, m, and s?
2. Rewrite the formula in terms of acceleration (solve the formula for acceleration).
3. If you apply a force of 15 Newton to an object (mass = 3 kg), what is its acceleration?

### Solution

$$1. F = ma = (kg)\left(\frac{m}{s^2}\right) = \frac{kg \cdot m}{s^2} \quad \text{Insert the units for mass (kg) and acceleration (m/s}^2\text{)}$$

$$2. F = ma \quad \text{Write down formula}$$

$$\frac{F}{m} = \frac{\cancel{m}a}{\cancel{m}} \quad \text{Divide both sides by } m \text{ and cancel } m \text{ on the right side}$$

$$\frac{F}{m} = a \quad \text{Rewrite} \quad \text{or} \quad a = \frac{F}{m}$$

$$3. a = \frac{F}{m} = \frac{15N}{3kg} = 5 \frac{m}{s^2} \quad \text{The acceleration is } 5 \text{ m/s}^2.$$

### PROBLEMS

7. You apply a force of 20N to an object with mass of 10kg. What is its acceleration?

8. A force of 150 Newton is applied to an object. It accelerates at  $2 \text{ m/s}^2$ . What is the mass of the object?

9. Find the area (A) of a triangle with a height (h) of 6 cm and a base (b) of 8cm. ( $A_{\text{TRIANGLE}} = \frac{1}{2}bh$ )

10. Find the base of a triangle with an area of 25cm<sup>2</sup> and a height of 5cm.

11. Find the radius (r) of a circle with an area (A) of 100cm<sup>2</sup>. ( $A_{\text{CIRCLE}} = \pi r^2$ )

12. Find the radius (r) of a circle with a circumference (C) of 25 cm. ( $C = 2\pi r$ )

13. The volume (V) of a rectangular prism is given by  $V = lwh$ , where l = length, w = width, and h = height of the prism.

a. Solve the equation for height (h). (Rearrange the equation that h is on one side and all the other terms are on the other side.

b. Solve the equation for width (w).



**14.** A rectangular swimming pool is 5 meters by 7 meters long. You want to build a fence that is 3 meters away from the perimeter of the swimming pool. If the fence sells for \$25 per meter, how much is the fence? It helps to draw a sketch.

**15.** A flower vase has a square base with each side 4 inches. The vase is 12 inches tall. The vase must be filled up halfway to keep the flowers healthy. How much water is needed? (Hint: Find the area of the base and then multiply the area by the height of the vase. That gives you the entire volume of the vase.)

16. John weighs 155 pounds. Represent his weight after he lost  $x$  pounds.

17. Jane is  $n$  years old now. Represent her age 4 years from now.

18. Saul is 10 years old now. Represent his age  $b$  years ago.

19. Barbara is 36 years old. If Don is  $n$  years younger than Barbara represent Don's age.

## Lesson 1.3 Represent Functions as Rules and Tables

### Vocabulary

A **function** consists of:

A **set** called the **domain** containing numbers called **inputs** and a **set** called the **range** containing numbers called **outputs**.

The **input** is called the **independent variable**.

The **output** is called the **dependent variable**. The **output** (dependent variable) is dependent on the value of the **input variable**.

### Example:

#### Identify the domain and range of a function

The input-output table represents the price of various lobsters at a fish market. Identify the domain and range of the function.

Input (pounds)	1.4	2.2	3.2	4.3	5.1	5.3
Output (dollars)	\$7.60	\$10.90	\$16.10	\$20.50	\$25.70	\$26.90

Solution: The domain is the set of inputs: 1.4, 2.2, 3.2, 4.3, 5.1, 5.3.

The range is the set of outputs: 7.60, 10.90, 16.10, 20.50, 25.70, 26.90.

### PROBLEMS

Identify the domain and range of the function.

1.

Input	1	3	5	7
Output	0	2	4	6

2.

Input	0	2	6	9
Output	0	1	3	5

3.

Input	5	7	15	17
Output	3	2	1	8

**Example 2      Make a table for a function**

The domain of the function  $y = x + 2$  is 0, 2, 5, 6. Make a table for the function then identify the range of the function.

x	0	2	5	6
$y=x+2$	$0+2=2$	$2+2=4$	$5+2=7$	$6+2=8$

The range of the function is 2, 4, 7, 8.

**PROBLEMS**

**Make a table for the function. Identify the range of the function.**

**1.  $y = 2x - 1$                       Domain = 0, 1, 3, 5**

x				
$y=2x-1$				

The range of the function is:

**2.  $y = -4x + 3$                       Domain = -2, 2, 4, 5**

x				
$y=-4x+3$				

The range of the function is:

**3.  $y = 0.5x - 3$                       Domain = 0, 1, 2, 3**

x				
$y=0.5x-3$				

The range of the function is:

**4.  $y = \frac{1}{2}x - 2$                       Domain = 2, 4, 8, 10**

x				
$y=\frac{1}{2}x-2$				

The range of the function is:

**Example 3 Write a rule for the function**

Input	2	4	6	8
Output	6	12	18	24

**Solution** Let  $x$  be the input and  $y$  be the output. Realize that the output is 3 times the input. Therefore, a rule for the function is  $y = 3x$ .

**PROBLEMS****Write a rule for the function**

1.

Input	3	6	7	8
Output	15	30	35	40

2.

Input	2	4	6	8
Output	3	5	7	9

3.

Input	1	2	3	4
Output	0	1	2	3

4.

Input	1	2	3	4
Output	4	5	6	7

## Lesson 1.4 Represent Functions as Graphs

### Example 1 Graph a function

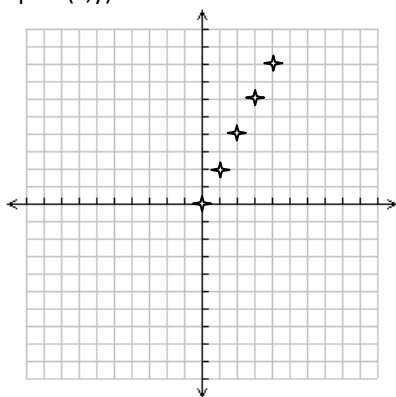
Graph the function  $y=2x$  with domain 0, 1, 2, 3, and 4.

Solution

Step 1: Make an input-output table.

x	0	1	2	3	4
y	0	2	4	6	8

Step 2: Plot a point for each ordered pair (x,y).



### PROBLEMS

Graph the function

1.  $y = 3x$  Domain: 0, 1, 2, 3

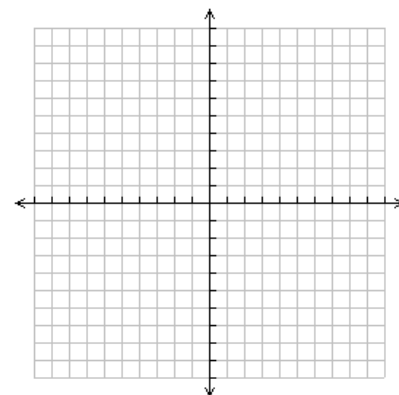
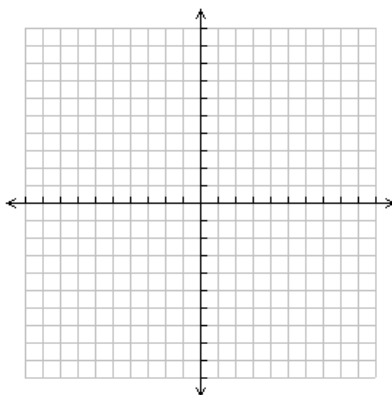
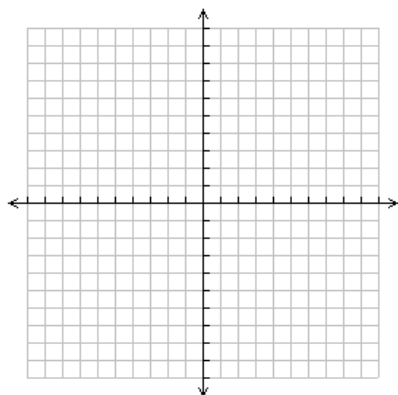
2.  $y = \frac{1}{2}x$  Domain: 0, 2, 4, 6

3.  $y = 2x - 2$  Domain: 0, 1, 2, 3

x	0	1	2	3
y				

x	0	2	4	6
y				

x	0	1	2	3
y				



**Lesson 1.5      Graph Using Intercepts**

**Vocabulary**      The x-coordinate of a point where the graph crosses the x-axis is the x-intercept.

                         The y-coordinate of a point where the graph crosses the y-axis is the y-intercept.

**Example 1      Find the intercepts of a graph of an equation.**

**Find the x-intercept and the y-intercept of the graph  $4x - 8y = 24$**

**Solution:**      To find the x intercept, replace y with a 0 and solve for x

$$4x - 8(0) = 24$$

$$\text{Solve for } x = 6$$

To find the y intercept, replace x with a 0 and solve for y

$$4(0) - 8y = 24$$

$$\text{Solve for } y = -3$$

Therefore, the x-intercept is 6. The y-intercept is -3.

**PROBLEMS**

**Find the x-intercept and the y-intercept of the graph**

1.       $-5x + 7y = 35$

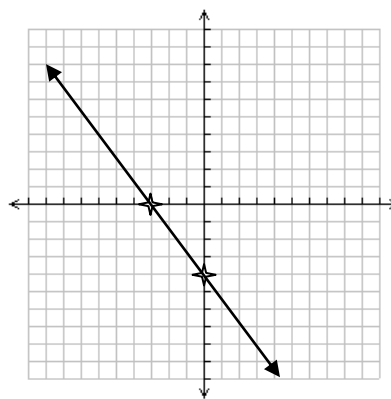
2.       $3x - 2y = -6$

3.       $y = \frac{1}{4}x + 5$

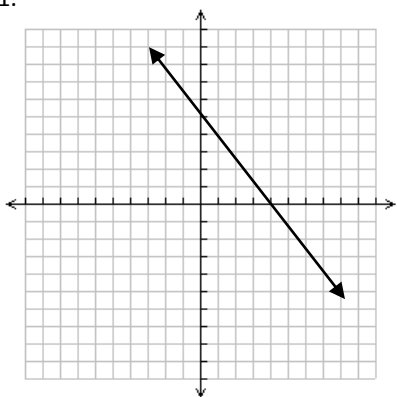
**Example 2** Use a graph to find the intercepts**Identify the x-intercept and the y-intercept of the graph**

Solution: To find the x-intercept, identify the point where the line crosses the x-axis. **The x-intercept is -3.**

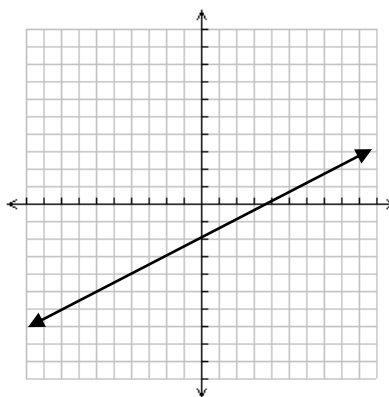
To find the y-intercept, identify the point where the line crosses the y-axis. **The y-intercept is -4.**

**PROBLEMS:** Identify the x-intercept and the y-intercept of the graph

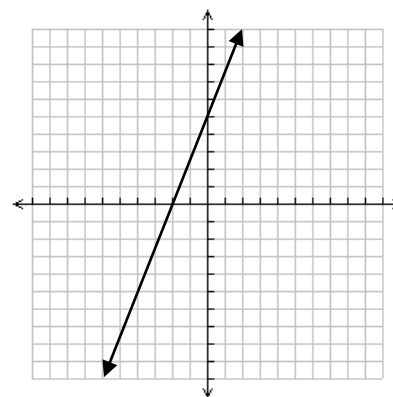
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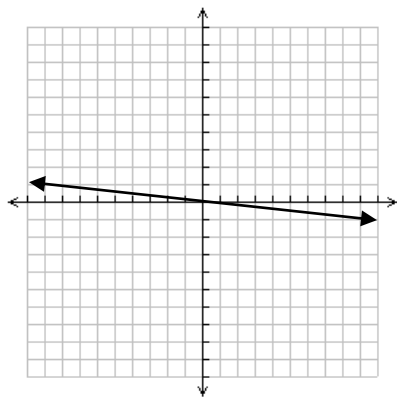
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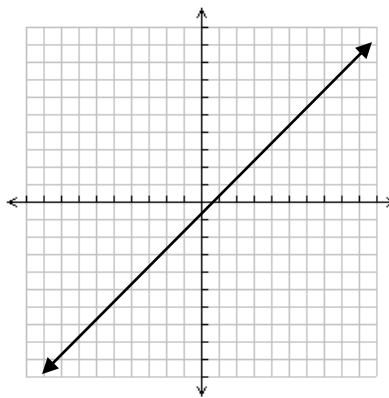
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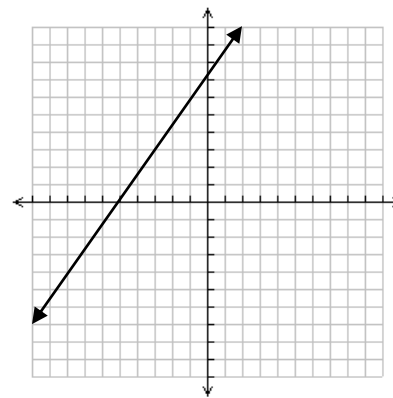
4.



5.



6.



**Example 3 Use intercepts to graph an equation****Graph  $2x + 3y = 6$ . Label the points where the line crosses the axes.****Solution:**Step 1: Find the intercepts: To find the y-intercept, set  $x = 0$  and solve for y.

$$2(0) + 3y = 6$$

$$\text{Solve for } y = 2$$

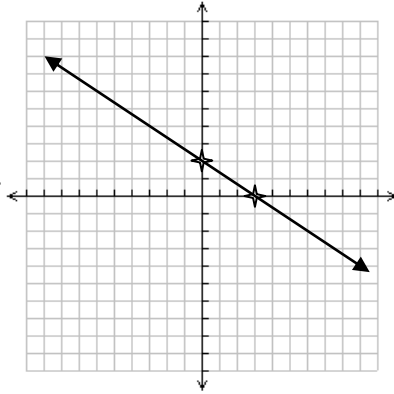
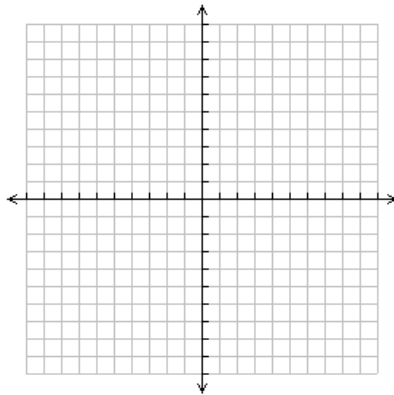
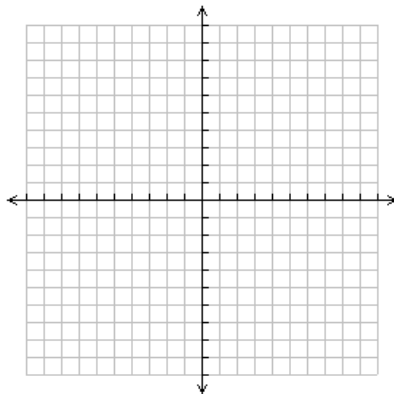
To find the x-intercept, set  $y = 0$  and solve for x.

$$2x + 3(0) = 6$$

$$\text{Solve for } x = 3$$

Step 2: Plot the points (intercepts) on the corresponding axes.

Step 3: Connect the points by drawing a line through them.

**1. Graph  $-3x + 2y = 6$ . Label the points where the line crosses the axes.****2. Graph  $5x - 4y = 20$ . Label the points where the line crosses the axes.**



## Lesson 1.6 Find the Slope and Rate of Change

### Vocabulary

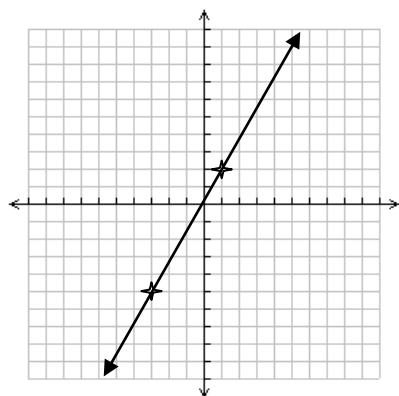
The **slope** of a line is the ratio of the vertical change (rise) to the horizontal change (run) between any two points of the line. **SLOPE = RISE/RUN**

### Example 1 Find a positive slope

Find the slope of the line shown.

#### Solution

Pick two convenient points on the line (that fall on a lattice point). Count the boxes (units) you move up. Count the boxes (units) you move to the right. Divide the number of boxes (units) you moved up by the number of boxes (units) you moved to the right to get the slope.

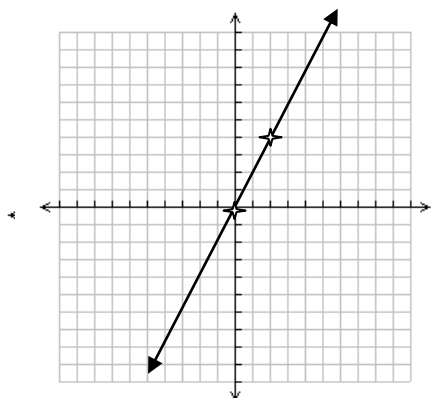


Pick the points (1,2) and (-3,-5). You start at the lower point and move 7 boxes (units) up. Then you move 4 Boxes (units) to the right. The slope is  $\frac{7}{4}$ .

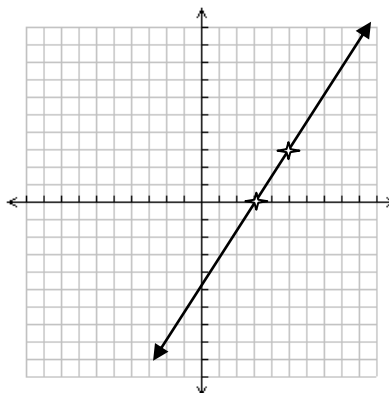
**PROBLEMS:** Find the slope of the line shown.

**SLOPE = RISE/RUN**

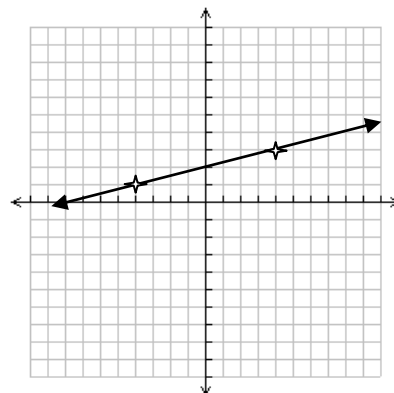
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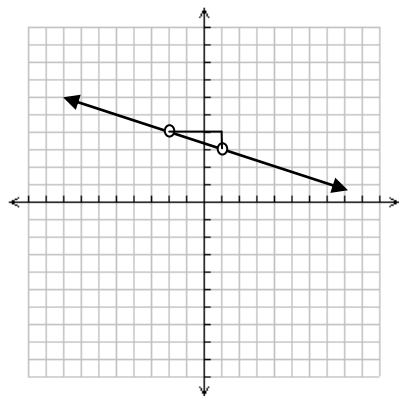


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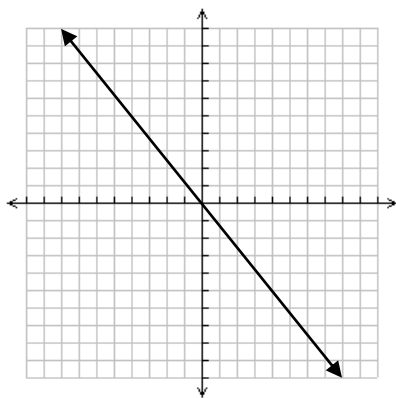
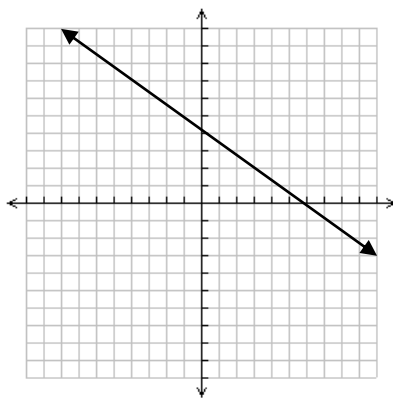
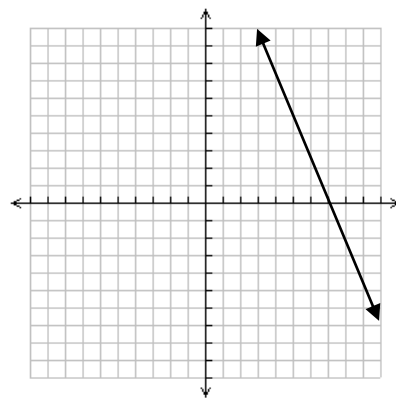


**Example 2 Find a negative slope****Find the slope of the line shown.****Solution**

Pick two convenient points on the line (that fall on a lattice point). Count the boxes (units) you move up. Count the boxes (units) you move to the left. Divide the number of boxes (units) you moved up by the number of boxes (units) you moved to the left to get the slope.



Pick the points (1,3) and (-2,4). You start at the lower point and move 1 box (units) up. Then you move 3 Boxes (units) to the left. The slope is  $-\frac{1}{3}$ .

**PROBLEMS: Find the slope of the line shown.****1.****2.****3.**

**Lesson 1.7      Graph Using Slope-Intercept****Vocabulary**

A linear equation (straight line relating  $x$  and  $y$ ) of the form  $y = mx + b$  is written in **slope-intercept form**. The letter  **$m$**  stands for the slope (how steep the line is) and the letter  **$b$**  for the  $y$ -intercept (where the line crosses the  $y$ -axis).

Two **parallel** lines will not intersect each other and have the same slope.

Two lines are **perpendicular** to each other if they intersect each other at  $90^\circ$ . Their slopes are negative reciprocals. For example if a line has a slope of 4 then a line perpendicular to that line will have a slope of  $-\frac{1}{4}$  (which is the negative reciprocal of 4).

**Example 1      Identify the slope and the  $y$ -intercept**

a.  $y = \frac{1}{4}x + 5$

Solution: The equation is in the form  $y = mx + b$ . The slope is  $\frac{1}{4}$ . The  $y$ -intercept is 5.

b.  $-5x + 6y = 12$

Solution: The equation is not in the form  $y = mx + b$ .

Rewrite the equation in slope-intercept form by solving for  $y$ .

$$-5x + 6y = 12$$

$$6y = 5x + 12 \quad \text{Add } 5x \text{ to each side}$$

$$y = \frac{5}{6}x + 2 \quad \text{Divide each side by } 6$$

The slope is  $\frac{5}{6}$ . The  $y$ -intercept is 2.

**PROBLEMS:** Find the slope and  $y$ -intercept of the line shown.

1.  $y = 5x - 6$

2.  $y = \frac{1}{2}x + 3$

3.  $y = -6x + 4$

4.  $y = -8x$

5.  $2x + 3y = 6$

6.  $y = 7 + 3x$

**Example 2: Graph an equation using slope intercept form**

Graph the equation  $2x + y = 3$ .

**Solution:**

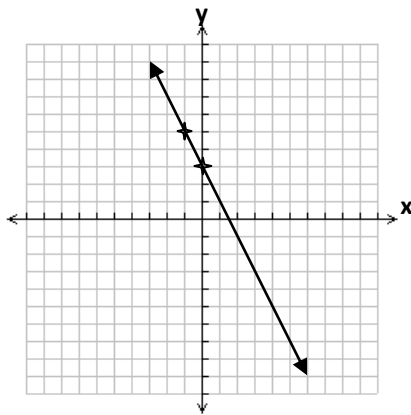
**Step 1: Rewrite the equation in slope-intercept form:**  $y = -2x + 3$

**Step 2: Identify the slope and the y-intercept:** slope: -2      y-intercept: 3

**Step 3: Mark the point (0,3) as y-intercept on the y-axis.**

**Step 4: From the y-intercept draw a slope of -2. -2 can be rewritten as  $-2 = \frac{2}{-1}$ .** Therefore, you rise 2 and run -1.

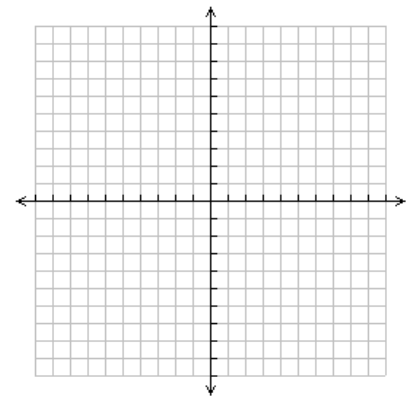
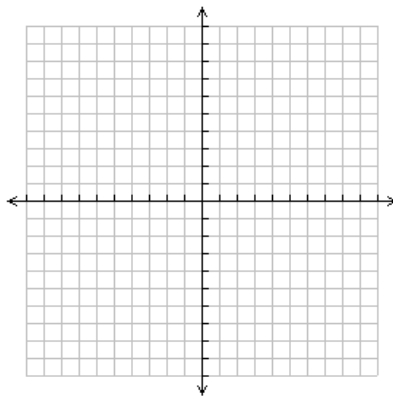
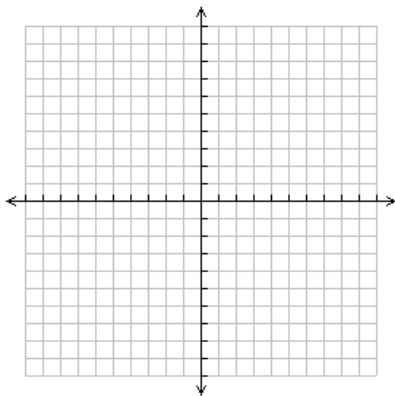
That means from the y-intercept of 3 you go up 2 (in the positive y-direction) and then 1 to the left (in the negative x-direction).

**PROBLEMS: Graph an equation using slope intercept form**

1.  $y = 5x - 6$

2.  $y = \frac{1}{2}x + 3$

3.  $2y + 6x = 4$



**Example 3: Determine the negative reciprocal of a number.**

a. 6 The negative reciprocal of 6 is  $-\frac{1}{6}$

b. -2 The negative reciprocal of -2 is  $\frac{1}{2}$

c.  $-\frac{3}{2}$  The negative reciprocal of  $-\frac{3}{2}$  is  $\frac{2}{3}$

**PROBLEMS:** Find the negative reciprocal of each numbers

1. 7

2.  $\frac{1}{2}$

3.  $-\frac{6}{7}$

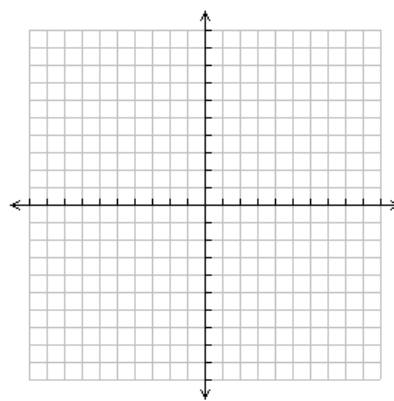
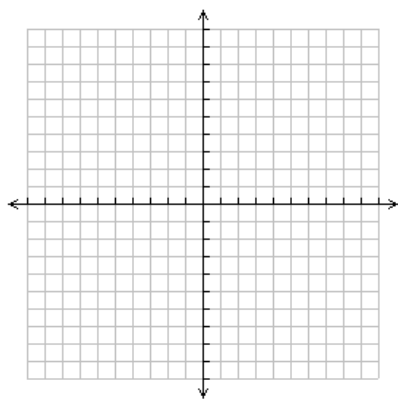
**Graph both equations on the same graph. Which pair is perpendicular? Which pair is parallel?**

4.  $y = 3x + 1$

$2y - 6x = 6$

5.  $y = \frac{1}{2}x - 2$

$2y + 4x = 2$



**Lesson 1.8      Graphing Linear Functions****Vocabulary**

A typical linear function is in the form  **$y = mx + b$** . By naming a function  $f$ , we can write the function using **function notation:  $f(x) = mx + b$** .

A **family of functions** is a group of showing similar characteristics.

Two **parallel** lines will not intersect each other and have the same a slope.

The most basic function of a linear function is called the **parent linear** function and has the form:  
 **$f(x) = x$** .

**Example 1**      What is the value of the function  $f(x) = -3x + 5$  when  $x = -2$ ?

**Solution:**

$$f(x) = -3x + 5$$

Write the original equation

$$f(-2) = -3(-2) + 5$$

Substitute  $-2$  for  $x$

$$= 11$$

Solve

**PROBLEMS:**      Evaluate the function for the given value of  $x$

1.  $f(x) = 2x + 3$  ,  $x = 2$

2.  $f(x) = -\frac{3}{5}x + 5$  ,  $x = -2$

3.  $g(x) = 2.8x - 5$  ,  $x = 0$

4.  $g(x) = \frac{4}{5}x - 3$  ,  $x = 5$

5.  $f(x) = 2.8x - 5$  ,  $x = 3$

6.  $f(x) = 2.25x + 6$  ,  $x = 4$

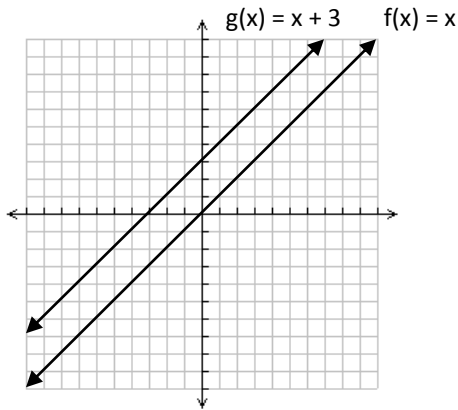
**Example 2****Comparing graphs with the graph  $f(x) = x$** 

Graph the function. Compare the graph with the graph  $f(x) = x$ .

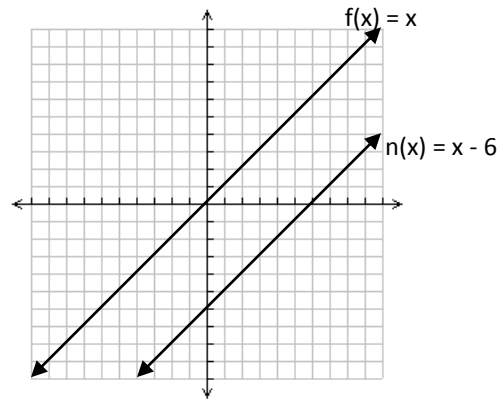
a.  $g(x) = x + 3$

b.  $n(x) = x - 5$

Graph the functions.



The graph  $g(x) = x + 3$  is shifted up by 3 units of the function  $f$ .



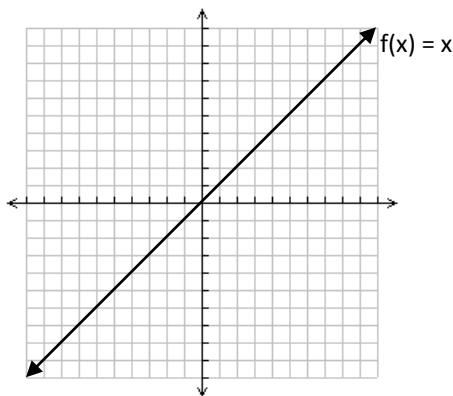
The graph  $n(x) = x - 6$  is shifted down by 6 units of the function  $f$ .

**PROBLEMS:** Compare the graph of  $g(x)$  with the graph  $f(x) = x$ .

1.  $g(x) = x + 2$

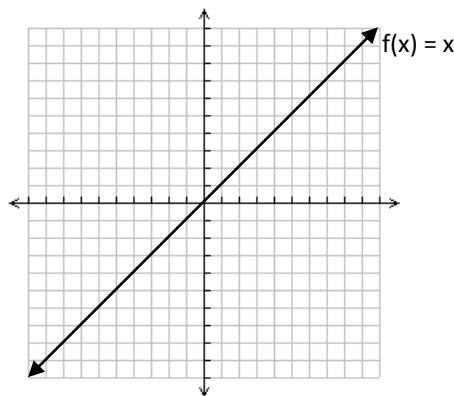
2.  $g(x) = x - 2$

3.  $g(x) = x + 6$



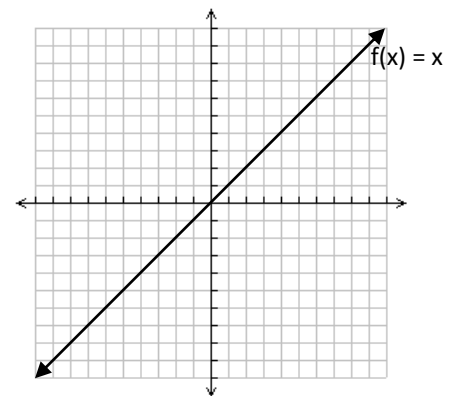
$g(x)$  is shifted up/down

by \_\_\_\_\_ units



$g(x)$  is shifted up/down

by \_\_\_\_\_ units



$g(x)$  is shifted up/down

by \_\_\_\_\_ units

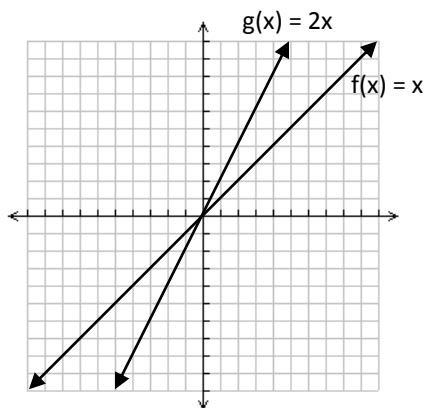
**Example 3****Comparing graphs with the graph  $f(x) = x$** 

**Graph the function. Compare the graph with the graph  $f(x) = x$ .**

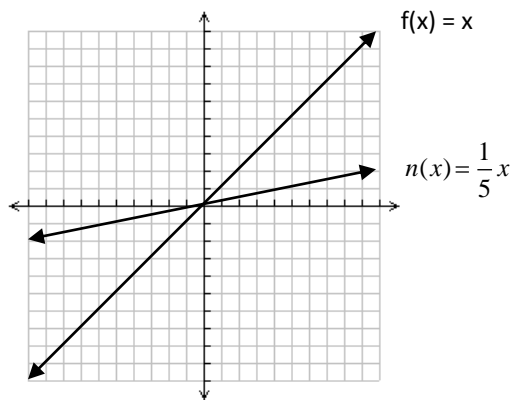
a.  $g(x) = 2x$

b.  $n(x) = \frac{1}{5}x$

**Solution:** Graph the functions.



The graph of  $g(x) = 2x$  is steeper than the original function  $f(x) = x$ .

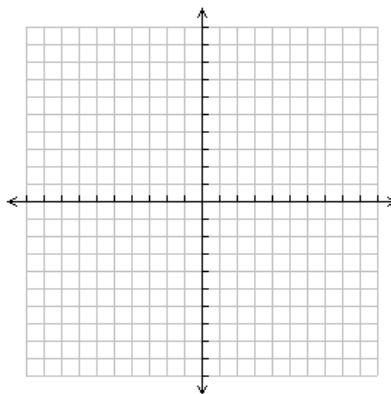
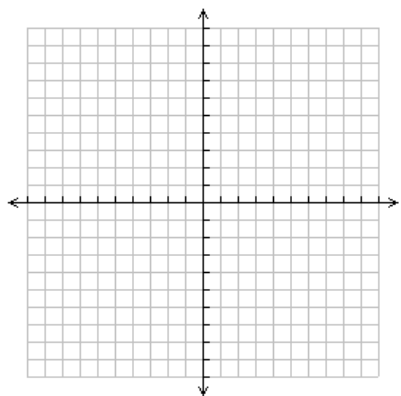


The graph of  $n(x) = \frac{1}{5}x$  is flatter than the original function  $f(x) = x$ .

**PROBLEMS:** Graph the function. Compare the graph with the graph  $f(x) = x$ .

a.  $g(x) = 3x$

b.  $n(x) = \frac{1}{3}x$





**Lesson 1.9****Predict with Linear Models****Vocabulary**

A line that best represents a trend in data or points is called the **best-fitting line**.

Using a line or its equation to approximate values between two known points is called **linear interpolation**.

Using a line or its equation to approximate values outside the range of two known points is called **linear extrapolation**.

**Example 1 Find the equation of the best fitting line.**

You are given following data:

X	-1	0	1	2	3	4
y	6	2	-2	-3	-6	-10

Find the equation of the best-fitting line for the data.

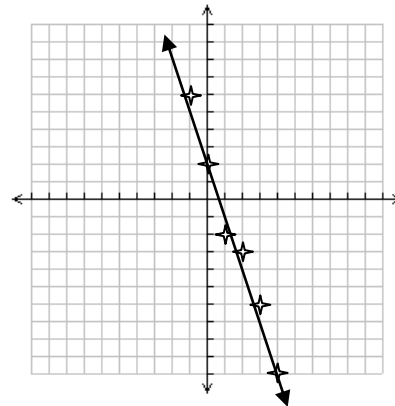
**Solution:**

Step 1: Draw a **straight** line representing the points.

Step 2: Find the equation of the line.

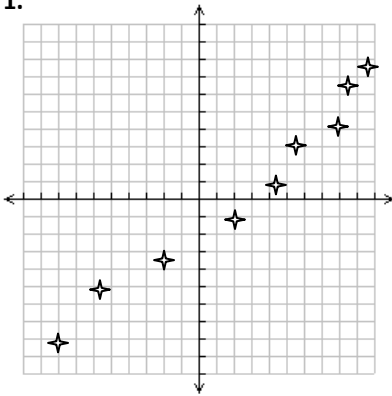
The y-intercept is 2 and the slope is -3.

Therefore, the equation of the line is  $y = -3x + 2$

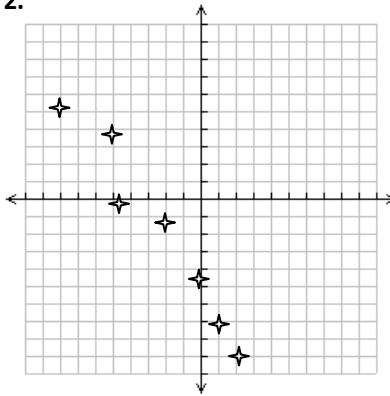


**PROBLEMS:** Find the equation of the best fitting line.

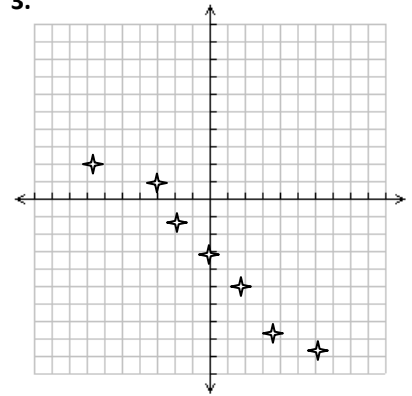
1.



2.

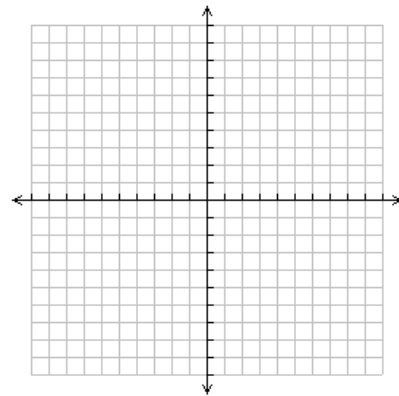


3.



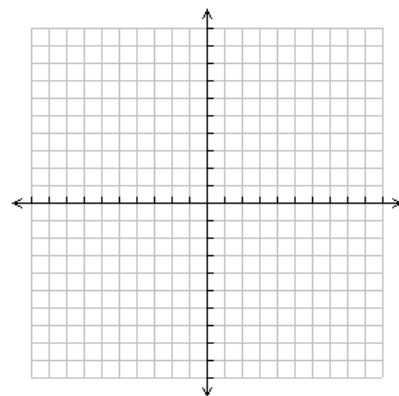
4. Make a scatter plot of the data. Then find the equation of the best fitting line.

x	-3	-2	0	1	3	4
y	-8	-4	-2	0	2	6



5. Make a scatter plot of the data. Then find the equation of the best fitting line.

x	-3	-2	0	2	3	4
y	3	2	-1	0	-1	-2



Lesson 1.10

Create Equations

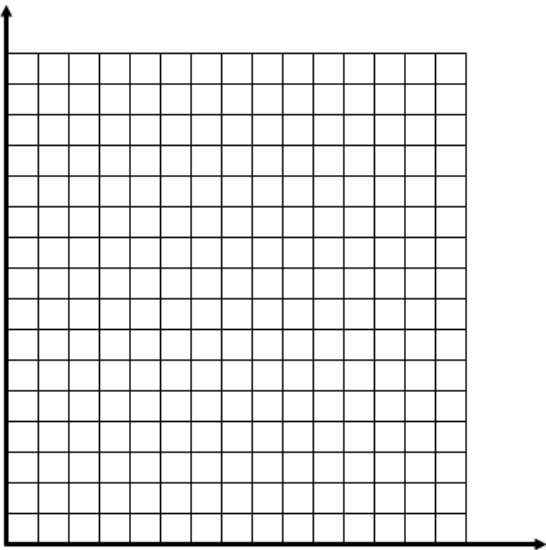
Jamie has a candle which is 16 centimeters tall. Each hour it burns makes the candle lose 2.5 centimeters in height. Jamie also has a shorter candle which is 12 centimeters tall. Each hour it burns the candle loses 1.5 centimeter in height.

Jamie wants to determine whether these two candles ever reach the same height at the same time if allowed to burn the same length of time.

Help Jamie to finish the table then put the info of the table into the graph below. What do you conclude?

Time (hours)	16 cm candle, Height (cm)	12 cm candle, Height (cm)
0	16	12
1	13.5	10.5
2		
3		
4		
5		
6		
7		

Graph: Candle Height on the y-axis, time on the x-axis.



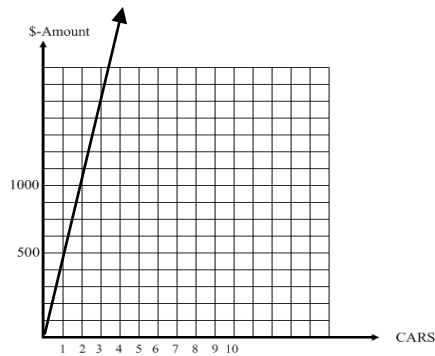
Conclusion:

Can you come up with an equation for each candle?

**Example 1** Find an equation which represents the problem. Jenny receives \$500 for each car she sells. Let  $x$  be the number of cars and  $y$  the total \$-amount.

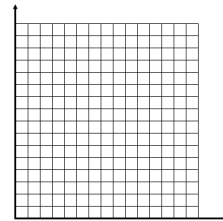
Then graph the equation.

**Solution:** The equation is  $y = 500x$ . The graph is shown below.

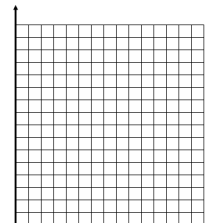


**PROBLEMS:** Find an equation which represents the problem. Then graph the equation.

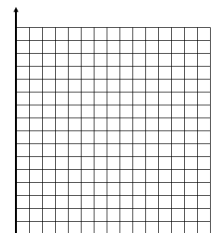
1. You earn \$7.50 per hour. Let  $x$  be the number of hours you work and  $y$  the total \$-amount.



2. You drive 60 miles per hour. Let  $x$  be the number of hours and  $y$  the total mileage.



3. You run a mile in 7 minutes. Let  $x$  be the miles and  $y$  the minutes.



## Lesson 1.11 Exponential Growth Functions

**Goal:** Graph and use exponential growth functions.

An **exponential function** has the form  $y = ab^x$ , where  $a \neq 0$  and the base  $b$  is a positive number other than 1. If  $a > 0$  and  $b > 0$ , then the function  $y = ab^x$  is an **exponential growth function** and  $b$  is called the growth factor.

Remember: A **negative exponent** “flips” the number:  $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$        $\frac{1}{3^{-3}} = 3^3 = 27$

An **asymptote** is a line that a graph approaches more and more closely (but never touches).

The **end behavior** of a function is the behavior of the graph as it approaches positive or negative infinity.

**Example 1** Graph  $y = 4^x$ . Analyze the graph.

**Solution:**

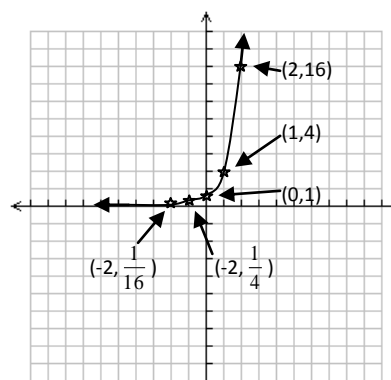
Step 1: Make a table of the graph

x	-2	-1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

Step 2: Plot the points from the table

Step 3: Draw a smooth curve through the points and continue to the left and the right to approximate the end behavior of the graph.

Step 4: Examine and analyze the graph.

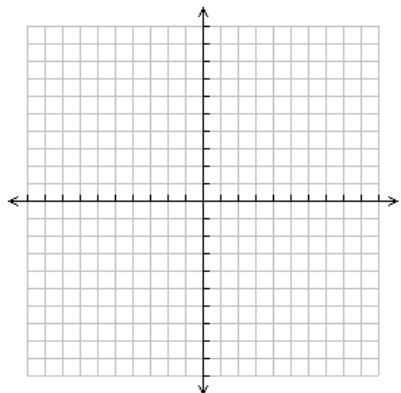


Examine and analyze the graph: As the value of  $x$  approaches negative infinity ( $-\infty$ ) the value of the function ( $y$ ) approaches but never reaches 0. As the value of  $x$  approaches positive infinity ( $+\infty$ ) the value of the function ( $y$ ) approaches  $+\infty$ .

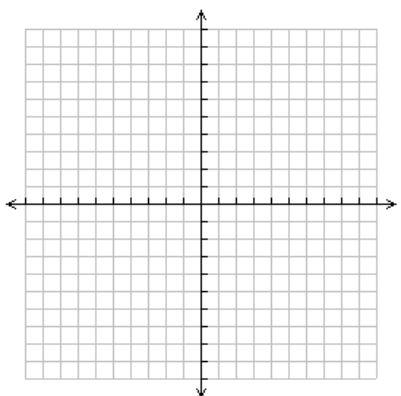
**PROBLEMS**

- a. Graph the function showing a chart.
- b. State the end behavior (as  $x$  approaches positive infinity,  $y$  approaches ....).
- c. State asymptote(s). The asymptote is a line the function approaches but never touches.
- d. If the asymptote is not  $y=0$ , why?

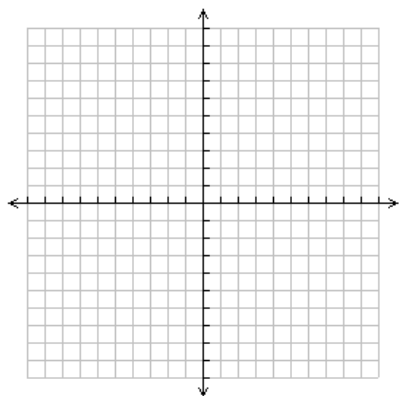
1.  $y = \left(\frac{4}{3}\right)^x$



2.  $y = \left(\frac{1}{2}\right)2^x$



3.  $y = 2^{x+1} - 5$



## Lesson 1.12 Exponential Decay Functions

**Goal:** Graph and use exponential decay functions.

An **exponential function** has the form  $y = ab^x$ , where  $a \neq 0$  and the base  $b$  is a positive number other than 1. If  $a > 0$  and  $0 < b < 1$ , then the function  $y = ab^x$  is an **exponential decay function** and  $b$  is called the decay factor.

An **asymptote** is a line that a graph approaches more and more closely (but never touches).

The **end behavior** of a function is the behavior of the graph as it approaches positive or negative infinity.

**Example 1** Graph  $y = \left(\frac{1}{3}\right)^x$ . Analyze the graph.

**Solution:**

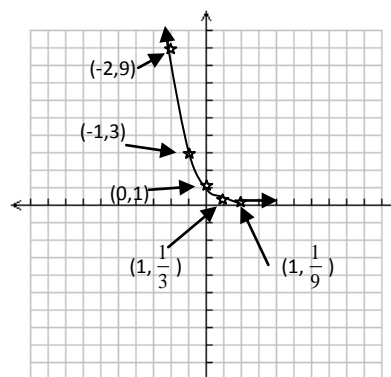
x	-2	-1	0	1	2
y	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$

Step 1: Make a table of the graph

Step 2: Plot the points from the table

Step 3: Draw a smooth curve through the points and continue to the left and the right to approximate the end behavior of the graph.

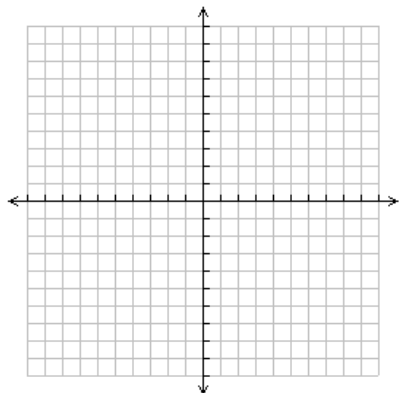
Step 4: Examine and analyze the graph.



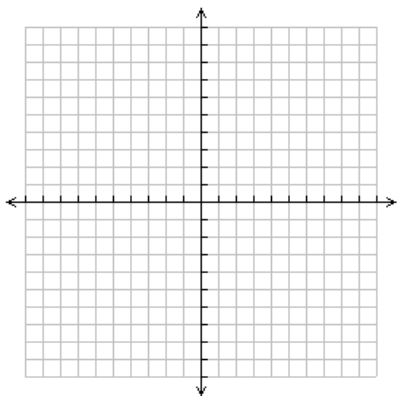
Examine and analyze the graph: As the value of  $x$  approaches negative infinity ( $-\infty$ ) the value of the function ( $y$ ) approaches positive infinity ( $+\infty$ ). As the value of  $x$  approaches positive infinity ( $+\infty$ ) the value of the function ( $y$ ) approaches but never reaches zero.

**PROBLEMS****a. Graph the function showing a chart.****b. State the end behavior (as  $x$  approaches positive infinity,  $y$  approaches ....).****c. State asymptote(s). The asymptote is a line the function approaches but never touches.****d. If the asymptote is not  $y=0$ , why?**

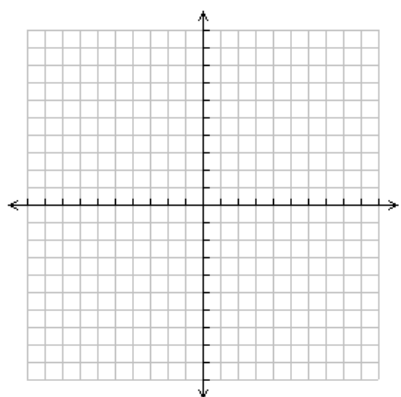
**1.**  $y = \left(\frac{3}{4}\right)^x$



**2.**  $y = 2\left(\frac{1}{4}\right)^x$



**3.**  $y = \left(\frac{1}{3}\right)^{(x+1)} - 3$





4. **Exponential decay**: You buy a car for \$21,500. The value of the car decreases every year by 12% (= depreciation rate). What is the value of the car after 5 years? The model to be used to solve this problem is:

$$\text{Value}(\$) = \text{Initial Price}(1 - \text{depreciation rate})^{\text{years}}$$

5. **Exponential growth**: You are 20 years old and receive \$10,000 from a relative. Instead of spending the money you chose to invest the money in the stock market by buying a mutual fund. Historically the stock market had an annual return of 10%. That means your capital invested grew by 10% per year. If historical returns apply to the future, how much money will you have in the mutual fund by the time you retire at 65? Use the following model:

$$\text{Value}(\$) = \text{Initial Investment}(1 + \text{annual return})^{\text{years}}$$

6. Combine problems 5 and 6: You have \$21,500 to buy a new car. Instead of buying a new car you buy a used car for \$10,000 and invest the remainder in the stock market buy a real nice car in 10 years. If you expect a return of 12% for the stock market, how much can you spend on a new car in 10 years?