Introduction to Relations and Functions

4

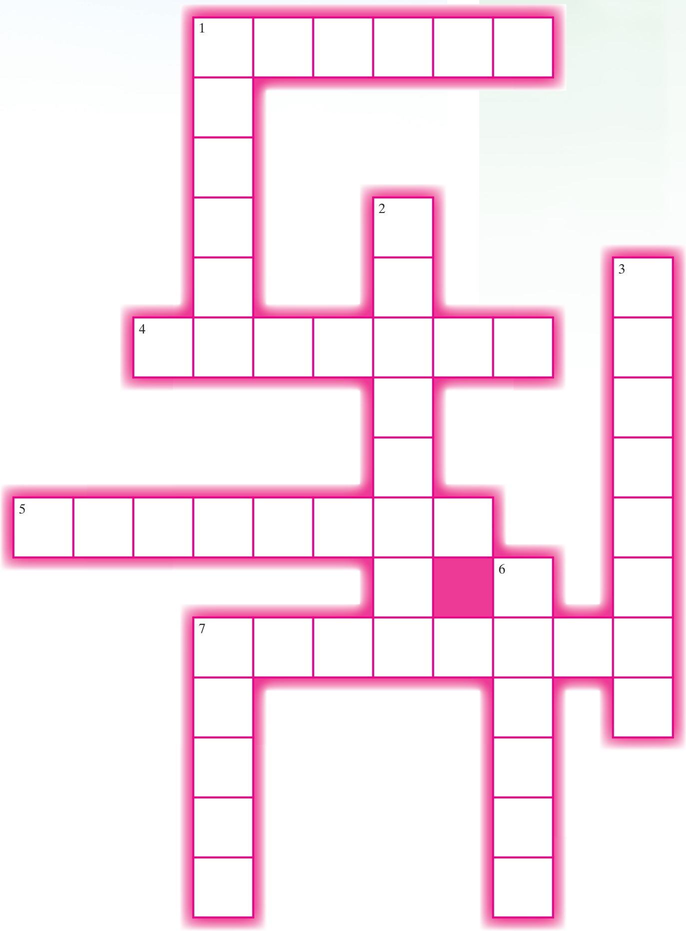
In this chapter we introduce the concept of a function. In general terms, a function defines how one variable depends on one or more other variables. The words in the puzzle are key terms found in this chapter.

Across

1. A type of variation such that as one variable increases, the other increases.
4. A type of variation such that as one variable increases, the other variable decreases.
5. A set of ordered pairs such that for every element in the domain, there corresponds exactly one element in the range.
7. A set of ordered pairs.

Down

1. The set of first coordinates of a set of ordered pairs.
2. The shape of the graph of a quadratic function.
3. A function whose graph is a horizontal line.
6. A function whose graph is a line that is not vertical or horizontal.
7. The set of second coordinates of a set of ordered pairs.



Section 4.1 Introduction to Relations

Concepts

1. Domain and Range of a Relation
2. Applications Involving Relations

1. Domain and Range of a Relation

In many naturally occurring phenomena, two variables may be linked by some type of relationship. For instance, an archeologist finds the bones of a woman at an excavation site. One of the bones is a femur. The femur is the large bone in the thigh attached to the knee and hip. Table 4-1 shows a correspondence between the length of a woman's femur and her height.

Table 4-1

Length of Femur (cm) x	Height (in.) y		Ordered Pair
45.5	65.5	→	(45.5, 65.5)
48.2	68.0	→	(48.2, 68.0)
41.8	62.2	→	(41.8, 62.2)
46.0	66.0	→	(46.0, 66.0)
50.4	70.0	→	(50.4, 70.0)

Each data point from Table 4-1 may be represented as an ordered pair. In this case, the first value represents the length of a woman's femur and the second, the woman's height. The set of ordered pairs {(45.5, 65.5), (48.2, 68.0), (41.8, 62.2), (46.0, 66.0), (50.4, 70.0)} defines a relation between femur length and height.

Definition of a Relation in *x* and *y*

Any set of ordered pairs (x, y) is called a **relation in *x* and *y***. Furthermore,

- The set of first components in the ordered pairs is called the **domain of the relation**.
- The set of second components in the ordered pairs is called the **range of the relation**.

Example 1 Finding the Domain and Range of a Relation

Find the domain and range of the relation linking the length of a woman's femur to her height {(45.5, 65.5), (48.2, 68.0), (41.8, 62.2), (46.0, 66.0), (50.4, 70.0)}.

Solution:

Domain:	{45.5, 48.2, 41.8, 46.0, 50.4}	Set of first coordinates
Range:	{65.5, 68.0, 62.2, 66.0, 70.0}	Set of second coordinates

Skill Practice

1. Find the domain and range of the relation.

$$\left\{(0, 0), (-8, 4), \left(\frac{1}{2}, 1\right), (-3, 4), (-8, 0)\right\}$$

Skill Practice Answers

1. Domain $\left\{0, -8, \frac{1}{2}, -3\right\}$, range $\{0, 4, 1\}$

The x- and y-components that constitute the ordered pairs in a relation do not need to be numerical. For example, Table 4-2 depicts five states in the United States and the corresponding number of representatives in the House of Representatives as of July 2005.

Table 4-2

State x	Number of Representatives y
Alabama	7
California	53
Colorado	7
Florida	25
Kansas	4

These data define a relation:

{(Alabama, 7), (California, 53), (Colorado, 7), (Florida, 25), (Kansas, 4)}

Example 2 Finding the Domain and Range of a Relation

Find the domain and range of the relation

{(Alabama, 7), (California, 53), (Colorado, 7), (Florida, 25), (Kansas, 4)}

Solution:

Domain: {Alabama, California, Colorado, Florida, Kansas}

Range: {7, 53, 25, 4} (*Note:* The element 7 is not listed twice.)

Skill Practice

2. The table gives the longevity for four types of animals. Write the ordered pairs (x, y) indicated by this relation, and state the domain and range.

Animal, x	Longevity (years), y
Bear	22.5
Cat	11
Deer	12.5
Dog	11

A relation may consist of a finite number of ordered pairs or an infinite number of ordered pairs. Furthermore, a relation may be defined by several different methods: by a list of ordered pairs, by a correspondence between the domain and range, by a graph, or by an equation.

Skill Practice Answers

2. {(Bear, 22.5), (Cat, 11), (Deer, 12.5), (Dog, 11)}; domain: {Bear, Cat, Deer, Dog}, range: {22.5, 11, 12.5}

- A relation may be defined as a set of ordered pairs.

$$\{(1, 2), (-3, 4), (1, -4), (3, 4)\}$$

- A relation may be defined by a correspondence (Figure 4-1). The corresponding ordered pairs are $\{(1, 2), (1, -4), (-3, 4), (3, 4)\}$.

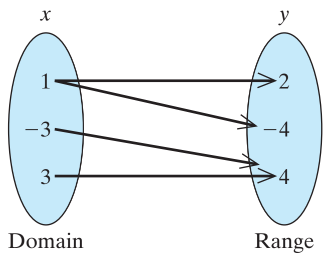


Figure 4-1

- A relation may be defined by a graph (Figure 4-2). The corresponding ordered pairs are $\{(1, 2), (-3, 4), (1, -4), (3, 4)\}$.

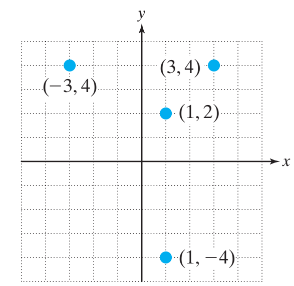


Figure 4-2

- A relation may be expressed by an equation such as $x = y^2$. The solutions to this equation define an infinite set of ordered pairs of the form $\{(x, y) | x = y^2\}$. The solutions can also be represented by a graph in a rectangular coordinate system (Figure 4-3).

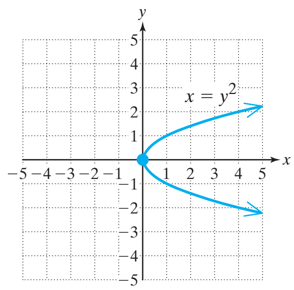


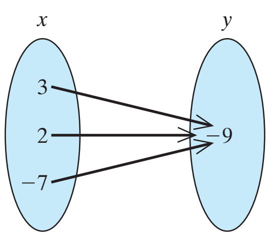
Figure 4-3

Example 3 Finding the Domain and Range of a Relation

Find the domain and range of the relations:

Solution:

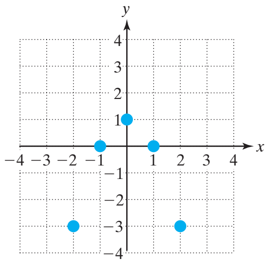
a.



Domain: $\{3, 2, -7\}$

Range: $\{-9\}$

b.

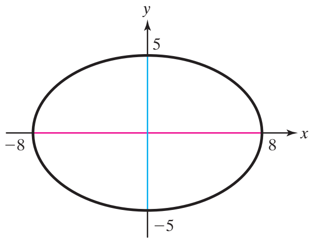


The domain elements are the x-coordinates of the points, and the range elements are the y-coordinates.

Domain: $\{-2, -1, 0, 1, 2\}$

Range: $\{-3, 0, 1\}$

c.

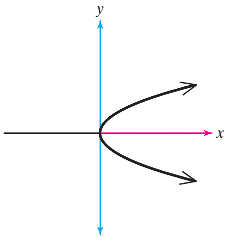


The domain consists of an infinite number of x-values extending from -8 to 8 (shown in red). The range consists of all y-values from -5 to 5 (shown in blue). Thus, the domain and range must be expressed in set-builder notation or in interval notation.

Domain: $\{x \mid x$ is a real number and $-8 \le x \le 8\}$ or $[-8, 8]$

Range: $\{y \mid y$ is a real number and $-5 \le y \le 5\}$ or $[-5, 5]$

d. $x = y^2$



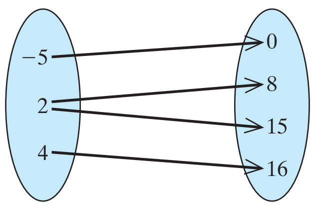
The arrows on the curve indicate that the graph extends infinitely far up and to the right and infinitely far down and to the right.

Domain: $\{x \mid x$ is a real number and $x \ge 0\}$ or $[0, \infty)$

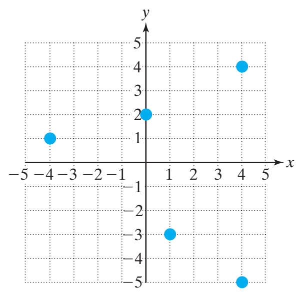
Range: $\{y \mid y$ is any real number$\}$ or $(-\infty, \infty)$

Skill Practice Find the domain and range of the relations.

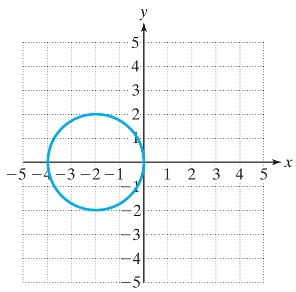
3.



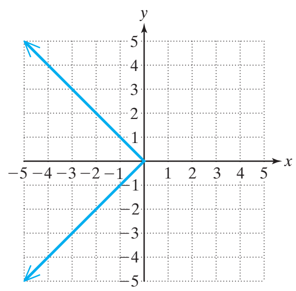
4.



5.



6. Find the domain and range of the relation $x = -|y|$ whose graph is shown here. Express the answer in interval notation.



Skill Practice Answers

3. Domain $\{-5, 2, 4\}$, range $\{0, 8, 15, 16\}$

4. Domain $\{-4, 0, 1, 4\}$, range $\{-5, -3, 1, 2, 4\}$

5. Domain: $\{x \mid x$ is a real number and $-4 \le x \le 0\}$ or $[-4, 0]$, range: $\{y \mid y$ is a real number and $-2 \le y \le 2\}$ or $[-2, 2]$

6. Domain: $(-\infty, 0]$, range: $(-\infty, \infty)$

2. Applications Involving Relations

Example 4 Analyzing a Relation

The data in Table 4-3 depict the length of a woman's femur and her corresponding height. Based on these data, a forensics specialist or archeologist can find a linear relationship between height y and femur length x:

$$y = 0.906x + 24.3 \qquad 40 \le x \le 55$$

From this type of relationship, the height of a woman can be inferred based on skeletal remains.

Table 4-3

Length of Femur (cm) x	Height (in.) y
45.5	65.5
48.2	68.0
41.8	62.2
46.0	66.0
50.4	70.0

a. Find the height of a woman whose femur is 46.0 cm.

b. Find the height of a woman whose femur is 51.0 cm.

c. Why is the domain restricted to $40 \le x \le 55$?

Solution:

a. $y = 0.906x + 24.3$
$= 0.906(46.0) + 24.3$ Substitute $x = 46.0$ cm.
$= 65.976$ The woman is approximately 66.0 in. tall.

b. $y = 0.906x + 24.3$
$= 0.906(51.0) + 24.3$ Substitute $x = 51.0$ cm.
$= 70.506$ The woman is approximately 70.5 in. tall.

c. The domain restricts femur length to values between 40 cm and 55 cm inclusive. These values are within the normal lengths for an adult female and are in the proximity of the observed data (Figure 4-4).

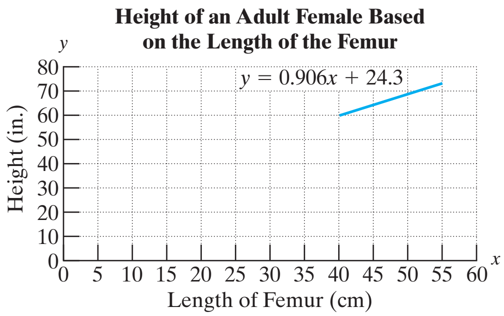


Figure 4-4

Skill Practice

7. The linear equation, $y = -0.014x + 64.5$, relates the weight of a car, x, (in pounds) to its gas mileage, y, (in mpg).

a. Find the gas mileage in miles per gallon for a car weighing 2550 lb.

b. Find the gas mileage for a car weighing 2850 lb.

Skill Practice Answers

7a. 28.8 mpg **b.** 24.6 mpg

Section 4.1 Practice Exercises

Study Skills Exercises

1. Compute your grade at this point. Are you earning the grade that you want? If not, maybe organizing a study group would help.

In a study group, check the activities that you might try to help you learn and understand the material.

_____ Quiz each other by asking each other questions.

_____ Practice teaching each other.

_____ Share and compare class notes.

_____ Support and encourage each other.

_____ Work together on exercises and sample problems.

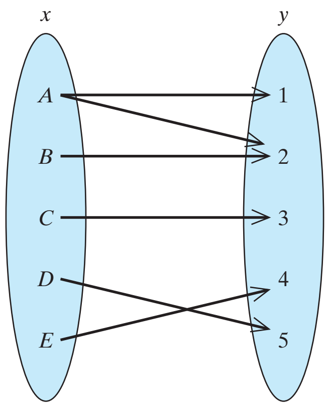
2. Define the key terms.

a. Relation in *x* and *y* **b. Domain of a relation** **c. Range of a relation**

Concept 1: Domain and Range of a Relation

For Exercises 3–6, write each relation as a set of ordered pairs.

3.



4.

State, x	Year of Statehood, y
Maine	1820
Nebraska	1823
Utah	1847
Hawaii	1959
Alaska	1959

5.

Memory Stick	Price
64 MB	\$37.99
128 MB	\$42.99
256 MB	\$49.99
512 MB	\$74.99

6.

x	y
0	3
-2	$\frac{1}{2}$
5	10
-7	1
-2	8
5	1

7. List the domain and range of Exercise 3.

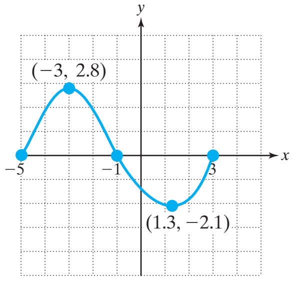
8. List the domain and range of Exercise 4.

9. List the domain and range of Exercise 5.

10. List the domain and range of Exercise 6.

For Exercises 11–24, find the domain and range of the relations. Use interval notation where appropriate.

11.



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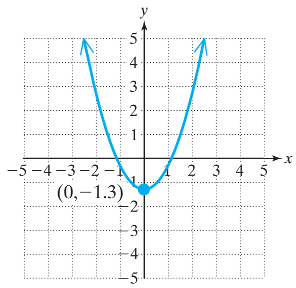

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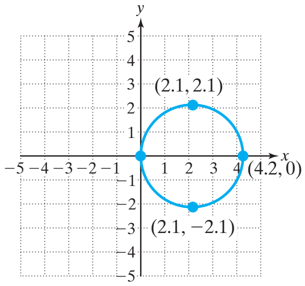

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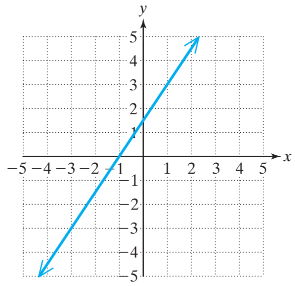

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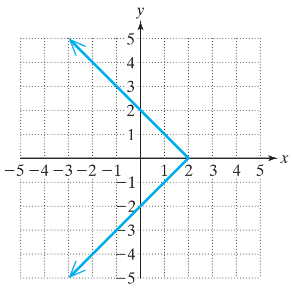
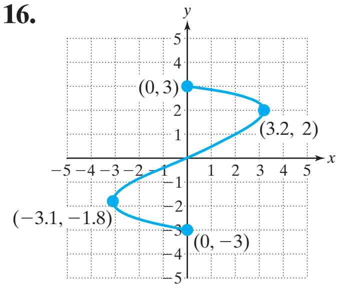
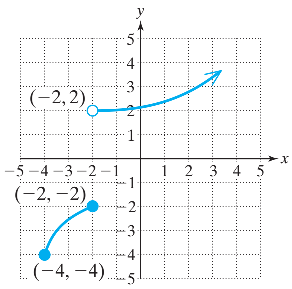


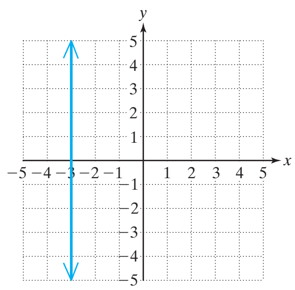
16.



17. *Hint:* The open circle indicates that the point is not included in the relation.



18.



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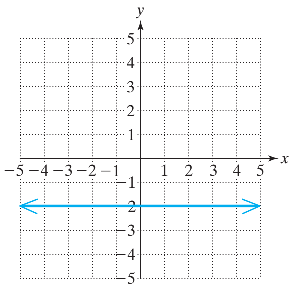

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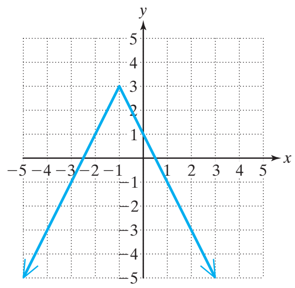
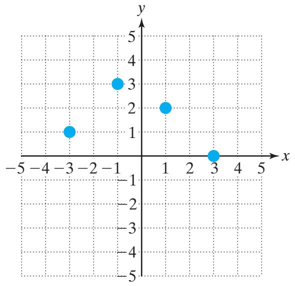
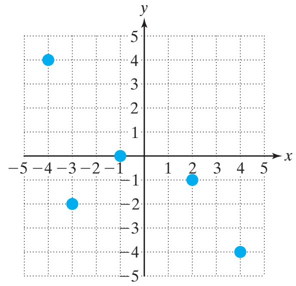


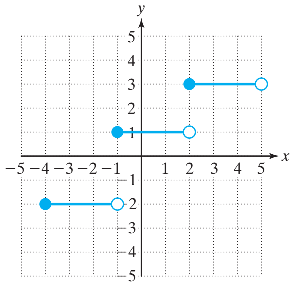
21.



22.



23.



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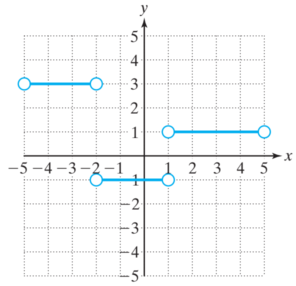


Concept 2: Applications Involving Relations

25. The table gives a relation between the month of the year and the average precipitation for that month for Miami, Florida.

a. What is the range element corresponding to April?

b. What is the range element corresponding to June?

c. Which element in the domain corresponds to the least value in the range?

d. Complete the ordered pair: (, 2.66)

e. Complete the ordered pair: (Sept.,)

f. What is the domain of this relation?

Month x	Precipitation (in.) y	Month x	Precipitation (in.) y
Jan.	2.01	July	5.70
Feb.	2.08	Aug.	7.58
Mar.	2.39	Sept.	7.63
Apr.	2.85	Oct.	5.64
May	6.21	Nov.	2.66
June	9.33	Dec.	1.83

Source: U.S. National Oceanic and Atmospheric Administration

26. The table gives a relation between a person's age and the person's maximum recommended heart rate.

a. What is the domain?

b. What is the range?

c. The range element 200 corresponds to what element in the domain?

d. Complete the ordered pair: (50,)

e. Complete the ordered pair: (, 190)

Age (years) x	Maximum Recommended Heart Rate (Beats per Minute) y
20	200
30	190
40	180
50	170
60	160

27. The population of Canada, y, (in millions) can be approximated by the relation $y = 0.146x + 31$, where x represents the number of years since 2000.

a. Approximate the population of Canada in the year 2006.

b. In what year will the population of Canada reach approximately 32,752,000?

28. As of April 2006, the world record times for selected women's track and field events are shown in the table.

The women's world record time y (in seconds) required to run x meters can be approximated by the relation $y = -10.78 + 0.159x$.

a. Predict the time required for a 500-m race.

b. Use this model to predict the time for a 1000-m race. Is this value exactly the same as the data value given in the table? Explain.

Distance (m)	Time (sec)	Winner's Name and Country
100	10.49	Florence Griffith Joyner (United States)
200	21.34	Florence Griffith Joyner (United States)
400	47.60	Marita Koch (East Germany)
800	113.28	Jarmila Kratochvilova (Czechoslovakia)
1000	148.98	Svetlana Masterkova (Russia)
1500	230.46	Qu Yunxia (China)

Expanding Your Skills

29. **a.** Define a relation with four ordered pairs such that the first element of the ordered pair is the name of a friend and the second element is your friend's place of birth.

b. State the domain and range of this relation.

30. **a.** Define a relation with four ordered pairs such that the first element is a state and the second element is its capital.

b. State the domain and range of this relation.

31. Use a mathematical equation to define a relation whose second component y is 1 less than 2 times the first component x.

32. Use a mathematical equation to define a relation whose second component y is 3 more than the first component x.

33. Use a mathematical equation to define a relation whose second component is the square of the first component.

34. Use a mathematical equation to define a relation whose second component is one-fourth the first component.

Section 4.2 Introduction to Functions

Concepts

1. Definition of a Function
2. Vertical Line Test
3. Function Notation
4. Finding Function Values from a Graph
5. Domain of a Function

1. Definition of a Function

In this section we introduce a special type of relation called a function.

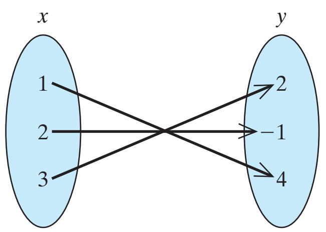
Definition of a Function

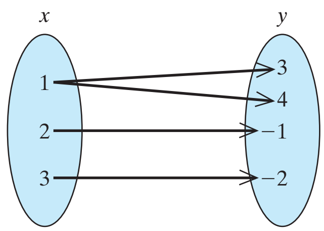
Given a relation in x and y, we say "y is a **function** of x" if for every element x in the domain, there corresponds exactly one element y in the range.

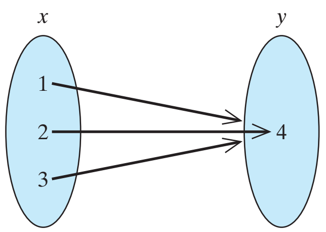
To understand the difference between a relation that is a function and a relation that is not a function, consider Example 1.

Example 1 Determining Whether a Relation Is a Function

Determine which of the relations define y as a function of x.

a.


b.


c.


Solution:

a. This relation is defined by the set of ordered pairs $\{(1, 4), (2, -1), (3, 2)\}$.

Notice that for each x in the domain there is only one corresponding y in the range. Therefore, this relation is a function.

When $x = 1$, there is only one possibility for y: $y = 4$

When $x = 2$, there is only one possibility for y: $y = -1$

When $x = 3$, there is only one possibility for y: $y = 2$

b. This relation is defined by the set of ordered pairs

same x

$$\{(1, 3), (1, 4), (2, -1), (3, -2)\}$$

different y-values

When $x = 1$, there are *two* possible range elements: $y = 3$ and $y = 4$. Therefore, this relation is *not* a function.

c. This relation is defined by the set of ordered pairs $\{(1, 4), (2, 4), (3, 4)\}$.

When $x = 1$, there is only one possibility for y: $y = 4$

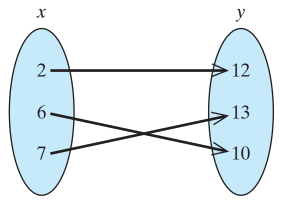
When $x = 2$, there is only one possibility for y: $y = 4$

When $x = 3$, there is only one possibility for y: $y = 4$

Because each value of x in the domain has only one corresponding y value, this relation is a function.

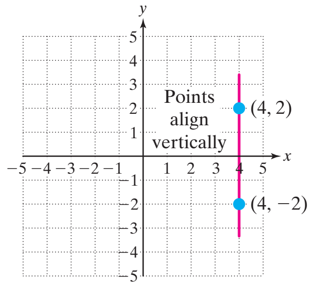
Skill Practice Determine if the relations define y as a function of x.

1.



2. $\{(4, 2), (-5, 4), (0, 0), (8, 4)\}$

3. $\{(-1, 6), (8, 9), (-1, 4), (-3, 10)\}$



2. Vertical Line Test

A relation that is not a function has at least one domain element x paired with more than one range value y. For example, the ordered pairs (4, 2) and (4, −2) do not constitute a function because two different y-values correspond to the same x. These two points are aligned vertically in the xy-plane, and a vertical line drawn through one point also intersects the other point. Thus, if a vertical line drawn through a graph of a relation intersects the graph in more than one point, the relation cannot be a function. This idea is stated formally as the **vertical line test**.

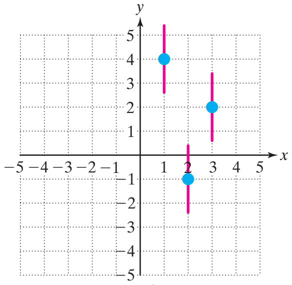
The Vertical Line Test

Consider a relation defined by a set of points (x, y) in a rectangular coordinate system. The graph defines y as a function of x if no vertical line intersects the graph in more than one point.

The vertical line test also implies that if any vertical line drawn through the graph of a relation intersects the relation in more than one point, then the relation does *not define y as a function of x*.

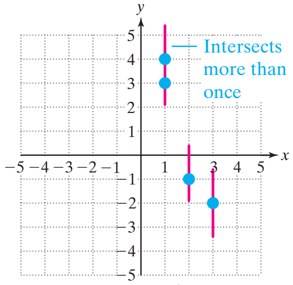
The vertical line test can be demonstrated by graphing the ordered pairs from the relations in Example 1.

a. $\{(1, 4), (2, -1), (3, 2)\}$



Function
No vertical line
intersects more than once.

b. $\{(1, 3), (1, 4), (2, -1), (3, -2)\}$



Not a Function
A vertical line intersects
in more than one point.

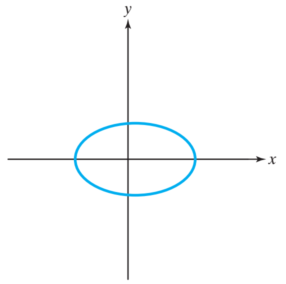
Skill Practice Answers

1. Yes **2.** Yes **3.** No

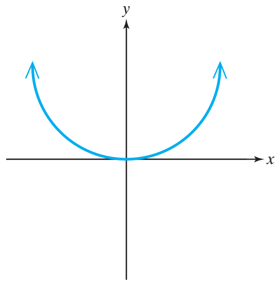
Example 2 Using the Vertical Line Test

Use the vertical line test to determine whether the following relations define y as a function of x.

a.

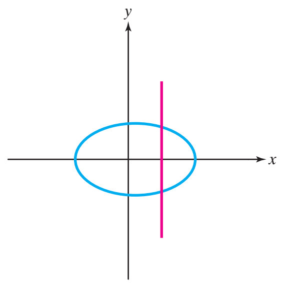


b.



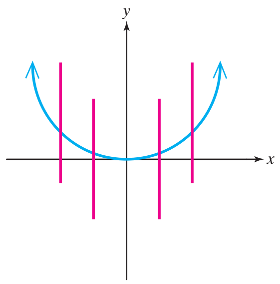
Solution:

a.



Not a Function
A vertical line intersects in more than one point.

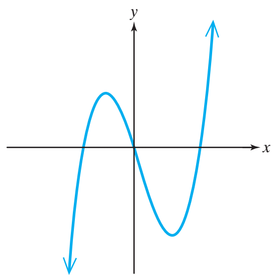
b.



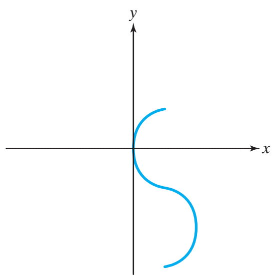
Function
No vertical line intersects in more than one point.

Skill Practice Use the vertical line test to determine whether the relations define y as a function of x.

4.



5.



3. Function Notation

A function is defined as a relation with the added restriction that each value in the domain must have only one corresponding y-value in the range. In mathematics, functions are often given by rules or equations to define the relationship between two or more variables. For example, the equation $y = 2x$ defines the set of ordered pairs such that the y-value is twice the x-value.

When a function is defined by an equation, we often use **function notation**. For example, the equation $y = 2x$ can be written in function notation as

Skill Practice Answers
4. Yes **5.** No

Avoiding Mistakes:

Be sure to note that $f(x)$ is *not* $f \cdot x$.

$f(x) = 2x$ where f is the name of the function, x is an input value from the domain of the function, and $f(x)$ is the function value (or y-value) corresponding to x

The notation $f(x)$ is read as "f of x" or "the value of the function f at x."

A function may be evaluated at different values of x by substituting x-values from the domain into the function. For example, to evaluate the function defined by $f(x) = 2x$ at $x = 5$, substitute $x = 5$ into the function.

TIP: The function value $f(5) = 10$ can be written as the ordered pair (5, 10).

$$f(x) = 2x$$
$$\downarrow \quad\quad \downarrow$$
$$f(5) = 2(5)$$
$$f(5) = 10$$

Thus, when $x = 5$, the corresponding function value is 10. We say "f of 5 is 10" or "f at 5 is 10."

The names of functions are often given by either lowercase or uppercase letters, such as f, g, h, p, K, and M.

Example 3 Evaluating a Function

Given the function defined by $g(x) = \frac{1}{2}x - 1$, find the function values.

a. $g(0)$ **b.** $g(2)$ **c.** $g(4)$ **d.** $g(-2)$

Solution:

a. $g(x) = \frac{1}{2}x - 1$

$g(0) = \frac{1}{2}(0) - 1$ Substitute 0 for x.

$= 0 - 1$

$= -1$ We say, "g of 0 is -1." This is equivalent to the ordered pair $(0, -1)$.

b. $g(x) = \frac{1}{2}x - 1$

$g(2) = \frac{1}{2}(2) - 1$

$= 1 - 1$

$= 0$ We say "g of 2 is 0." This is equivalent to the ordered pair (2, 0).

c. $g(x) = \frac{1}{2}x - 1$

$g(4) = \frac{1}{2}(4) - 1$

$= 2 - 1$

$= 1$ We say "g of 4 is 1." This is equivalent to the ordered pair (4, 1).

d. $g(x) = \frac{1}{2}x - 1$

$$g(-2) = \frac{1}{2}(-2) - 1$$
$$= -1 - 1$$
$$= -2$$

We say "g of -2 is -2." This is equivalent to the ordered pair $(-2, -2)$.

Notice that $g(0)$, $g(2)$, $g(4)$, and $g(-2)$ correspond to the ordered pairs $(0, -1)$, $(2, 0)$, $(4, 1)$, and $(-2, -2)$. In the graph, these points "line up." The graph of *all* ordered pairs defined by this function is a line with a slope of $\frac{1}{2}$ and y-intercept of $(0, -1)$ (Figure 4-5). This should not be surprising because the function defined by $g(x) = \frac{1}{2}x - 1$ is equivalent to $y = \frac{1}{2}x - 1$.

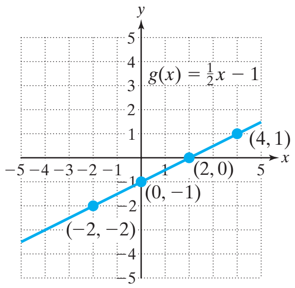


Figure 4-5

Skill Practice

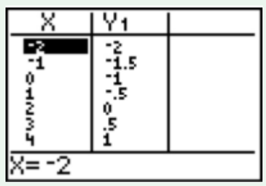
6. Given the function defined by $f(x) = -2x - 3$, find the function values.

a. $f(1)$ **b.** $f(0)$ **c.** $f(-3)$ **d.** $f\left(\frac{1}{2}\right)$

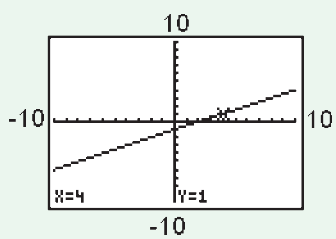
Calculator Connections

The values of $g(x)$ in Example 3 can be found using a *Table* feature.

$$Y_1 = \frac{1}{2}x - 1$$



Function values can also be evaluated by using a *Value* (or *Eval*) feature. The value of $g(4)$ is shown here.



A function may be evaluated at numerical values or at algebraic expressions, as shown in Example 4.

Skill Practice Answers

6a. -5 **b.** -3 **c.** 3 **d.** -4

Example 4 Evaluating Functions

Given the functions defined by $f(x) = x^2 - 2x$ and $g(x) = 3x + 5$, find the function values.

a. $f(t)$ **b.** $g(w + 4)$ **c.** $f(-t)$

Solution:

a. $f(x) = x^2 - 2x$

$f(t) = (t)^2 - 2(t)$ Substitute $x = t$ for all values of x in the function.

$= t^2 - 2t$ Simplify.

b. $g(x) = 3x + 5$

$g(w + 4) = 3(w + 4) + 5$ Substitute $x = w + 4$ for all values of x in the function.

$= 3w + 12 + 5$

$= 3w + 17$ Simplify.

c. $f(x) = x^2 - 2x$ Substitute $-t$ for x.

$f(-t) = (-t)^2 - 2(-t)$

$= t^2 + 2t$ Simplify.

Skill Practice

7. Given the function defined by $g(x) = 4x - 3$, find the function values.

a. $g(a)$ **b.** $g(x + h)$ **c.** $g(-x)$

4. Finding Function Values from a Graph

We can find function values by looking at a graph of the function. The value of $f(a)$ refers to the y-coordinate of a point with x-coordinate a.

Example 5 Finding Function Values from a Graph

Consider the function pictured in Figure 4-6.

a. Find $h(-1)$.

b. Find $h(2)$.

c. For what value of x is $h(x) = 3$?

d. For what values of x is $h(x) = 0$?

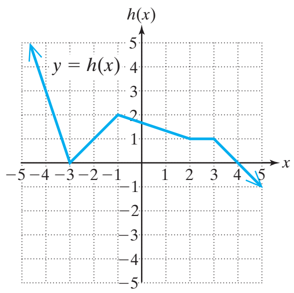


Figure 4-6

Solution:

a. $h(-1) = 2$ This corresponds to the ordered pair $(-1, 2)$.

b. $h(2) = 1$ This corresponds to the ordered pair $(2, 1)$.

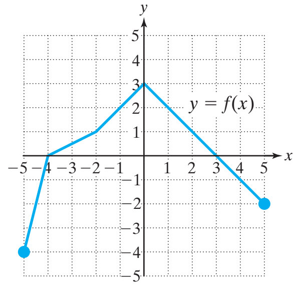
c. $h(x) = 3$ for $x = -4$ This corresponds to the ordered pair $(-4, 3)$.

d. $h(x) = 0$ for $x = -3$ and $x = 4$ T −3, 0)

Skill Practice Answers

7a. $4a - 3$ **b.** $4x + 4h - 3$ **c.** $-4x - 3$

Skill Practice Refer to the function graphed here.



8. Find $f(0)$.

9. Find $f(-2)$.

10. For what value(s) of x is $f(x) = 0$?

11. For what value(s) of x is $f(x) = -4$?

5. Domain of a Function

A function is a relation, and it is often necessary to determine its domain and range. Consider a function defined by the equation $y = f(x)$. The **domain** of f is the set of all x-values that when substituted into the function, produce a real number. The **range** of f is the set of all y-values corresponding to the values of x in the domain.

To find the domain of a function defined by $y = f(x)$, keep these guidelines in mind.

- Exclude values of x that make the denominator of a fraction zero.
- Exclude values of x that make a negative value within a square root.

Example 6 Finding the Domain of a Function

Find the domain of the functions. Write the answers in interval notation.

a. $f(x) = \dfrac{x + 7}{2x - 1}$ **b.** $h(x) = \dfrac{x - 4}{x^2 + 9}$

c. $k(t) = \sqrt{t + 4}$ **d.** $g(t) = t^2 - 3t$

Solution:

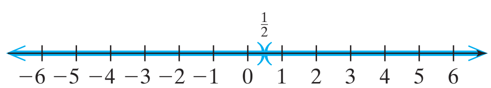
a. The function will be undefined when the denominator is zero, that is, when

$$2x - 1 = 0$$

$$2x = 1$$

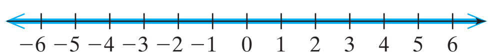
$$x = \frac{1}{2}$$ The value $x = \frac{1}{2}$ must be *excluded* from the domain.

Interval notation: $\left(-\infty, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, \infty\right)$



b. The quantity x^2 is greater than or equal to 0 for all real numbers x, and the number 9 is positive. Therefore, the sum $x^2 + 9$ must be *positive* for all real numbers x. The denominator of $h(x) = (x - 4)/(x^2 + 9)$ will never be zero; the domain is the set of all real numbers.

Interval notation: $(-\infty, \infty)$



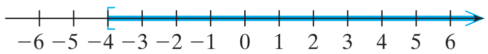
Skill Practice Answers

8. 3 **9.** 1
10. $x = -4$ and $x = 3$
11. $x = -5$

c. The function defined by $k(t) = \sqrt{t + 4}$ will not be a real number when $t + 4$ is negative; hence the domain is the set of all t-values that make the radicand *greater than or equal to zero:*

$$t + 4 \geq 0$$

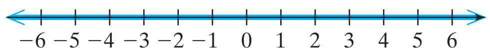
$$t \geq -4$$



Interval notation: $[-4, \infty)$

d. The function defined by $g(t) = t^2 - 3t$ has no restrictions on its domain because any real number substituted for t will produce a real number. The domain is the set of all real numbers.

Interval notation: $(-\infty, \infty)$



Skill Practice Write the domain of the functions in interval notation.

12. $f(x) = \dfrac{2x + 1}{x - 9}$ **13.** $p(x) = \dfrac{-5}{4x^2 + 1}$

14. $g(x) = \sqrt{x - 2}$ **15.** $h(x) = x + 6$

Skill Practice Answers

12. $(-\infty, 9) \cup (9, \infty)$
13. $(-\infty, \infty)$
14. $[2, \infty)$
15. $(-\infty, \infty)$

Section 4.2 Practice Exercises

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Study Skills Exercise

1. Define the key terms.

a. **Function** **b.** **Function notation** **c.** **Domain** **d.** **Range** **e.** **Vertical line test**

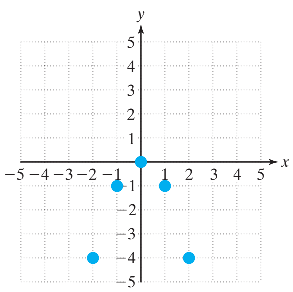
Review Exercises

For Exercises 2–4, **a.** write the relation as a set of ordered pairs, **b.** identify the domain, and **c.** identify the range.

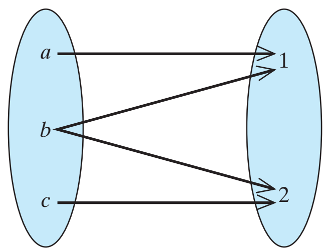
2.

Parent, x	Child, y
Kevin	Katie
Kevin	Kira
Kathleen	Katie
Kathleen	Kira

3.



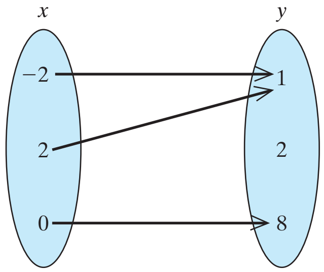
4.



Concept 1: Definition of a Function

For Exercises 5–10, determine if the relation defines y as a function of x.

5.



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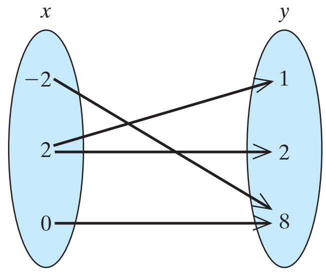

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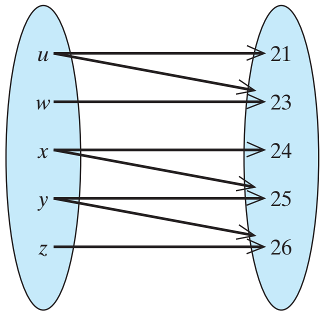

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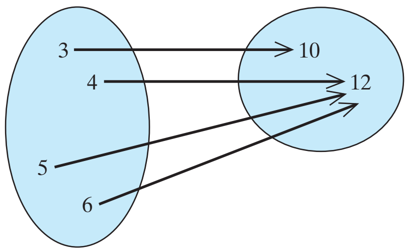


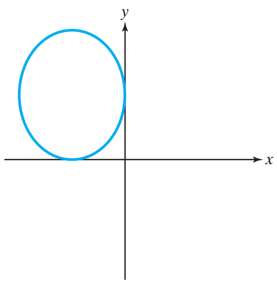
9. $\{(1, 2), (3, 4), (5, 4), (-9, 3)\}$

10. $\left\{(0, -1.1), \left(\frac{1}{2}, 8\right), (1.1, 8), \left(4, \frac{1}{2}\right)\right\}$

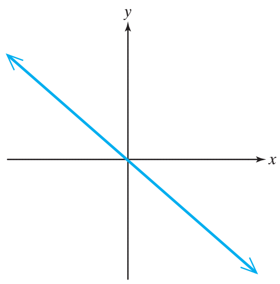
Concept 2: Vertical Line Test

For Exercises 11–16, use the vertical line test to determine whether the relation defines y as a function of x.

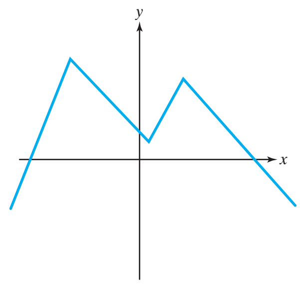
11.



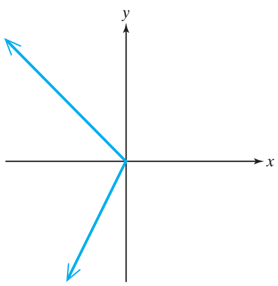
12.



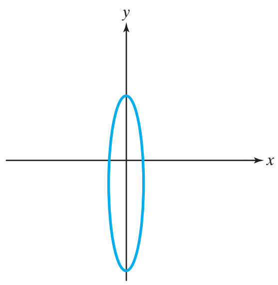
13.



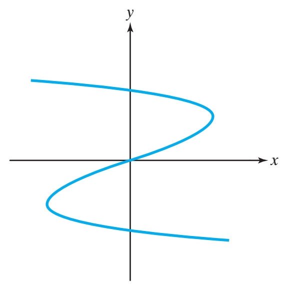
14.



15.



16.



Concept 3: Function Notation

Consider the functions defined by $f(x) = 6x - 2$, $g(x) = -x^2 - 4x + 1$, $h(x) = 7$, and $k(x) = |x - 2|$. For Exercises 17–48, find the following.

17. $g(2)$ **18.** $k(2)$ **19.** $g(0)$ **20.** $h(0)$

21. $k(0)$ **22.** $f(0)$ **23.** $f(t)$ **24.** $g(a)$

25. $h(u)$ **26.** $k(v)$ **27.** $g(-3)$ **28.** $h(-5)$

29. $k(-2)$ **30.** $f(-6)$ **31.** $f(x + 1)$ **32.** $h(x + 1)$

33. $g(2x)$ **34.** $k(x - 3)$ **35.** $g(-\pi)$ **36.** $g(x + h)$

37. $h(a + b)$ **38.** $f(x + h)$ **39.** $f(-a)$ **40.** $g(-b)$

41. $k(-c)$ **42.** $h(-x)$ **43.** $f\left(\frac{1}{2}\right)$ **44.** $g\left(\frac{1}{4}\right)$

45. $h\left(\frac{1}{7}\right)$ **46.** $k\left(\frac{3}{2}\right)$ **47.** $f(-2.8)$ **48.** $k(-5.4)$

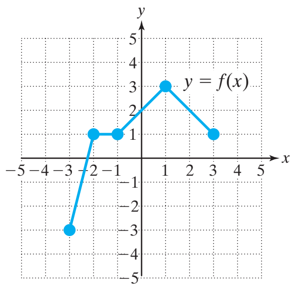
Consider the functions $p = \{(\frac{1}{2}, 6), (2, -7), (1, 0), (3, 2\pi)\}$ and $q = \{(6, 4), (2, -5), (\frac{3}{4}, \frac{1}{5}), (0, 9)\}$. For Exercises 49–56, find the function values.

49. $p(2)$ **50.** $p(1)$ **51.** $p(3)$ **52.** $p\left(\frac{1}{2}\right)$

53. $q(2)$ **54.** $q\left(\frac{3}{4}\right)$ **55.** $q(6)$ **56.** $q(0)$

Concept 4: Finding Function Values from a Graph

57. The graph of $y = f(x)$ is given.



a. Find $f(0)$.

b. Find $f(3)$.

c. Find $f(-2)$.

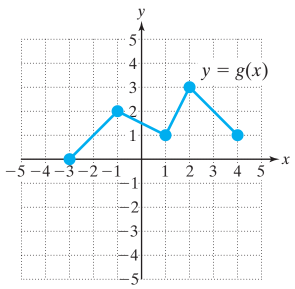
d. For what value(s) of x is $f(x) = -3$?

e. For what value(s) of x is $f(x) = 3$?

f. Write the domain of f.

g. Write the range of f.

58. The graph of $y = g(x)$ is given.



a. Find $g(-1)$.

b. Find $g(1)$.

c. Find $g(4)$.

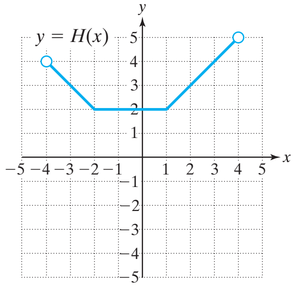
d. For what value(s) of x is $g(x) = 3$?

e. For what value(s) of x is $g(x) = 0$?

f. Write the domain of g.

g. Write the range of g.

59. The graph of $y = H(x)$ is given.



a. Find $H(-3)$.

b. Find $H(4)$.

c. Find $H(3)$.

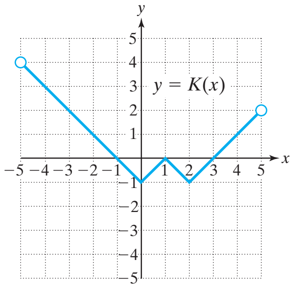
d. For what value(s) of x is $H(x) = 3$?

e. For what value(s) of x is $H(x) = 2$?

f. Write the domain of H.

g. Write the range of H.

60. The graph of $y = K(x)$ is given.



a. Find $K(0)$.

b. Find $K(-5)$.

c. Find $K(1)$.

d. For what value(s) of x is $K(x) = 0$?

e. For what value(s) of x is $K(x) = 3$?

f. Write the domain of K.

g. Write the range of K.

Concept 5: Domain of a Function

61. Explain how to determine the domain of the function defined by $f(x) = \dfrac{x + 6}{x - 2}$.

62. Explain how to determine the domain of the function defined by $g(x) = \sqrt{x - 3}$.

For Exercises 63–78, find the domain. Write the answers in interval notation.

63. $k(x) = \dfrac{x - 3}{x + 6}$

64. $m(x) = \dfrac{x - 1}{x - 4}$

65. $f(t) = \dfrac{5}{t}$

66. $g(t) = \dfrac{t - 7}{t}$

67. $h(p) = \dfrac{p - 4}{p^2 + 1}$

68. $n(p) = \dfrac{p + 8}{p^2 + 2}$

69. $h(t) = \sqrt{t + 7}$

70. $k(t) = \sqrt{t - 5}$

71. $f(a) = \sqrt{a - 3}$

72. $g(a) = \sqrt{a + 2}$

73. $m(x) = \sqrt{1 - 2x}$

74. $n(x) = \sqrt{12 - 6x}$

75. $p(t) = 2t^2 + t - 1$

76. $q(t) = t^3 + t - 1$

77. $f(x) = x + 6$

78. $g(x) = 8x - \pi$

Mixed Exercises

79. The height (in feet) of a ball that is dropped from an 80-ft building is given by $h(t) = -16t^2 + 80$, where t is time in seconds after the ball is dropped.

a. Find $h(1)$ and $h(1.5)$

b. Interpret the meaning of the function values found in part (a).

80. A ball is dropped from a 50-m building. The height (in meters) after t sec is given by $h(t) = -4.9t^2 + 50$.

a. Find $h(1)$ and $h(1.5)$.

b. Interpret the meaning of the function values found in part (a).

81. If Alicia rides a bike at an average of 11.5 mph, the distance that she rides can be represented by $d(t) = 11.5t$, where t is the time in hours.

a. Find $d(1)$ and $d(1.5)$.

b. Interpret the meaning of the function values found in part (a).

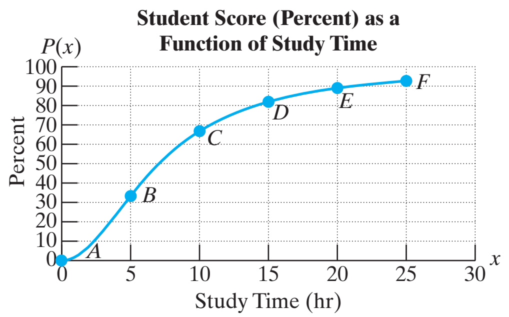
82. If Miguel walks at an average of 5.9 km/hr, the distance that he walks can be represented by $d(t) = 5.9t$, where t is the time in hours.

a. Find $d(1)$ and $d(2)$.

b. Interpret the meaning of the function values found in part (a).

83. Brian's score on an exam is a function of the number of hours he spends studying. The function defined by $P(x) = \dfrac{100x^2}{50 + x^2}$ $(x \geq 0)$ indicates that he will achieve a score of $P\%$ if he studies for x hours.

a. Evaluate $P(0)$, $P(5)$, $P(10)$, $P(15)$, $P(20)$, and $P(25)$. (Round to 1 decimal place.) Interpret $P(25)$ in the context of this problem.



b. Match the function values found in part (a) with the points A, B, C, D, E, and F on the graph.

Expanding Your Skills

For Exercises 84–85, find the domain. Write the answers in interval notation.

84. $q(x) = \dfrac{2}{\sqrt{x + 2}}$

85. $p(x) = \dfrac{8}{\sqrt{x - 4}}$

For Exercises 86–95, refer to the functions $y = f(x)$ and $y = g(x)$, defined as follows:

$$f = \{(-3, 5), (-7, -3), (-\tfrac{3}{2}, 4), (1.2, 5)\}$$

$$g = \{(0, 6), (2, 6), (6, 0), (1, 0)\}$$

86. Identify the domain of f.

87. Identify the range of f.

88. Identify the range of g.

89. Identify the domain of g.

90. For what value(s) of x is $f(x) = 5$?

91. For what value(s) of x is $f(x) = -3$?

92. For what value(s) of x is $g(x) = 0$?

93. For what value(s) of x is $g(x) = 6$?

94. Find $f(-7)$.

95. Find $g(0)$.

Graphing Calculator Exercises

96. Graph $k(t) = \sqrt{t-5}$. Use the graph to support your answer to Exercise 70.

97. Graph $h(t) = \sqrt{t+7}$. Use the graph to support your answer to Exercise 69.

98. **a.** Graph $h(t) = -4.9t^2 + 50$ on a viewing window defined by $0 \le t \le 3$ and $0 \le y \le 60$.

b. Use the graph to approximate the function at $t = 1$. Use these values to support your answer to Exercise 80.

99. **a.** Graph $h(t) = -16t^2 + 80$ on a viewing window defined by $0 \le t \le 2$ and $0 \le y \le 100$.

b. Use the graph to approximate the function at $t = 1$. Use these values to support your answer to Exercise 79.

Graphs of Functions

Section 4.3

Concepts

1. Linear and Constant Functions
2. Graphs of Basic Functions
3. Definition of a Quadratic Function
4. Finding the x- and y-Intercepts of a Function Defined by $y = f(x)$
5. Determining Intervals of Increasing, Decreasing, or Constant Behavior

1. Linear and Constant Functions

A function may be expressed as a mathematical equation that relates two or more variables. In this section, we will look at several elementary functions.

We know from Section 2.2 that an equation in the form $y = k$, where k is a constant, is a horizontal line. In function notation, this can be written as $f(x) = k$. For example, the function defined by $f(x) = 3$ is a horizontal line, as shown in Figure 4-7.

We say that a function defined by $f(x) = k$ is a constant function because for any value of x, the function value is constant.

An equation of the form $y = mx + b$ is represented graphically by a line with slope m and y-intercept $(0, b)$. In function notation, this can be written as $f(x) = mx + b$. A function in this form is called a linear function. For example, the function defined by $f(x) = 2x - 3$ is a linear function with slope $m = 2$ and y-intercept $(0, -3)$ (Figure 4-8).

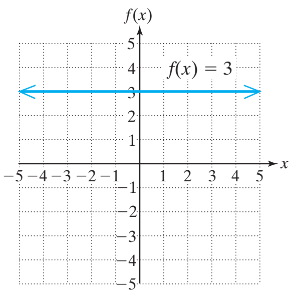


Figure 4-7

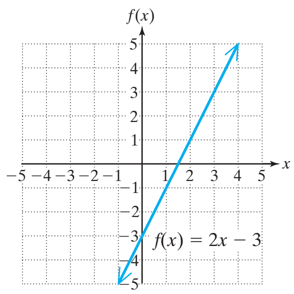


Figure 4-8

Definition of a Linear Function and a Constant Function

Let m and b represent real numbers such that $m \neq 0$. Then

A function that can be written in the form $f(x) = mx + b$ is a **linear function**.
A function that can be written in the form $f(x) = b$ is a **constant function**.

Note: The graphs of linear and constant functions are lines.

2. Graphs of Basic Functions

At this point, we are able to recognize the equations and graphs of linear and constant functions. In addition to linear and constant functions, the following equations define six basic functions that will be encountered in the study of algebra:

Equation		Function Notation
$y = x$		$f(x) = x$
$y = x^2$	equivalent function notation	$f(x) = x^2$
$y = x^3$	$\longrightarrow$	$f(x) = x^3$
$y = \lvert x \rvert$		$f(x) = \lvert x \rvert$
$y = \sqrt{x}$		$f(x) = \sqrt{x}$
$y = \dfrac{1}{x}$		$f(x) = \dfrac{1}{x}$

The graph of the function defined by $f(x) = x$ is linear, with slope $m = 1$ and y-intercept $(0, 0)$ (Figure 4-9).

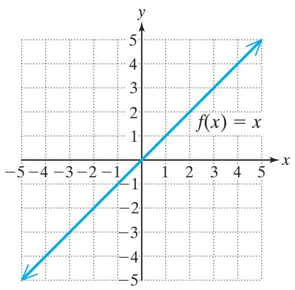


Figure 4-9

To determine the shapes of the other basic functions, we can plot several points to establish the pattern of the graph. Analyzing the equation itself may also provide insight to the domain, range, and shape of the function. To demonstrate this, we will graph $f(x) = x^2$ and $g(x) = \frac{1}{x}$.

Example 1 Graphing Basic Functions

Graph the functions defined by

a. $f(x) = x^2$ **b.** $g(x) = \dfrac{1}{x}$

Solution:

a. The domain of the function given by $f(x) = x^2$ (or equivalently $y = x^2$) is all real numbers.

To graph the function, choose arbitrary values of x within the domain of the function. Be sure to choose values of x that are positive and values that are negative to determine the behavior of the function to the right and left of the origin (Table 4-4). The graph of $f(x) = x^2$ is shown in Figure 4-10.

The function values are equated to the square of x, so $f(x)$ will always be greater than or equal to zero. Hence, the y-coordinates on the graph will never be negative. The range of the function is $\{y|y \text{ is a real number and } y \geq 0\}$. The arrows on each branch of the graph imply that the pattern continues indefinitely.

Table 4-4

x	$f(x) = x^2$
0	0
1	1
2	4
3	9
−1	1
−2	4
−3	9

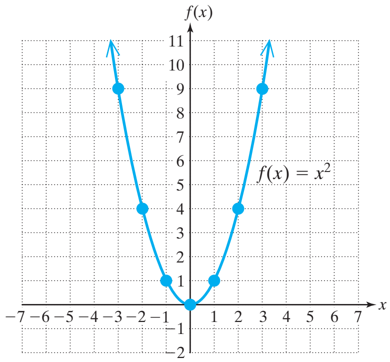


Figure 4-10

b. $g(x) = \dfrac{1}{x}$ Notice that $x = 0$ is not in the domain of the function. From the equation $y = \frac{1}{x}$, the y-values will be the reciprocal of the x-values. The graph defined by $g(x) = \frac{1}{x}$ is shown in Figure 4-11.

x	$g(x) = \dfrac{1}{x}$
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
−1	−1
−2	$-\frac{1}{2}$
−3	$-\frac{1}{3}$

x	$g(x) = \dfrac{1}{x}$
$\frac{1}{2}$	2
$\frac{1}{3}$	3
$\frac{1}{4}$	4
$-\frac{1}{2}$	−2
$-\frac{1}{3}$	−3
$-\frac{1}{4}$	−4

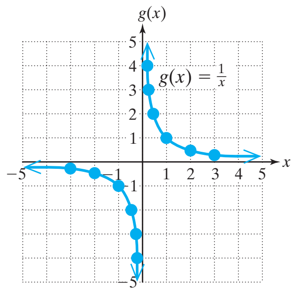
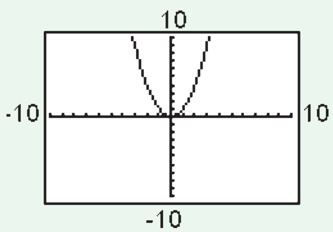


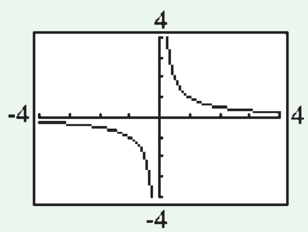
Figure 4-11

Notice that as x increases, the y-value decreases and gets closer to zero. In fact, as x approaches $-\infty$ or ∞, the graph gets closer to the x-axis. In this case, the x-axis is called a *horizontal asymptote*. Similarly, the graph of the function approaches the y-axis as x gets close to zero. In this case, the y-axis is called a *vertical asymptote*.

Calculator Connections

The graphs of the functions defined by $f(x) = x^2$ and $g(x) = \frac{1}{x}$ are shown in the following calculator displays.



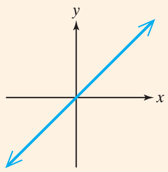
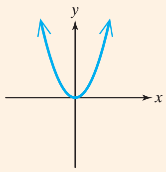
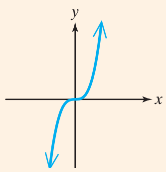
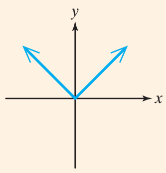
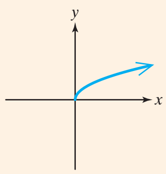
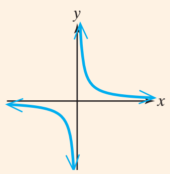


Skill Practice

1. Graph $f(x) = -x^2$ by first making a table of points.
2. Graph $h(x) = |x| - 1$ by first making a table of points.

For your reference, we have provided the graphs of six basic functions in the following table.

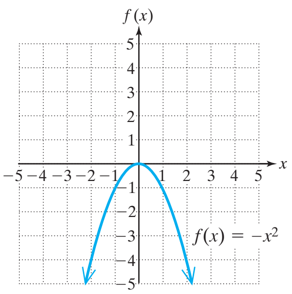
Summary of Six Basic Functions and Their Graphs

Function	Graph	Domain and Range
1. $f(x) = x$		Domain $(-\infty, \infty)$ Range $(-\infty, \infty)$
2. $y = x^2$		Domain $(-\infty, \infty)$ Range $[0, \infty)$
3. $y = x^3$		Domain $(-\infty, \infty)$ Range $(-\infty, \infty)$
4. $f(x) = \|x\|$		Domain $(-\infty, \infty)$ Range $[0, \infty)$
5. $y = \sqrt{x}$		Domain $[0, \infty)$ Range $[0, \infty)$
6. $y = \dfrac{1}{x}$		Domain $(-\infty, 0) \cup (0, \infty)$ Range $(-\infty, 0) \cup (0, \infty)$

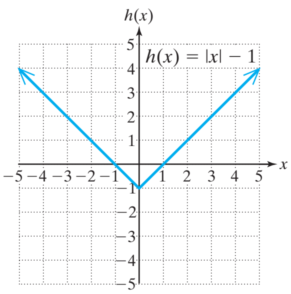
The shapes of these six graphs will be developed in the homework exercises. These functions are used often in the study of algebra. Therefore, we recommend that you associate an equation with its graph and commit each to memory.

Skill Practice Answers

1.



2.



3. Definition of a Quadratic Function

In Example 1 we graphed the function defined by $f(x) = x^2$ by plotting points. This function belongs to a special category called **quadratic functions**. A quadratic function can be written in the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. The graph of a quadratic function is in the shape of a **parabola**. The leading coefficient, a, determines the direction of the parabola.

If $a > 0$, then the parabola opens upward, for example, $f(x) = x^2$. The minimum point on a parabola opening upward is called the vertex (Figure 4-12).

If $a < 0$, then the parabola opens downward, for example, $f(x) = -x^2$. The maximum point on a parabola opening downward is called the vertex (Figure 4-13).

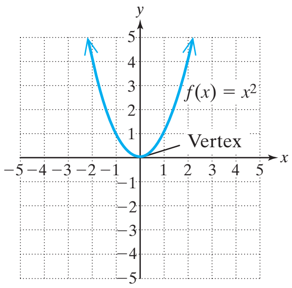


Figure 4-12

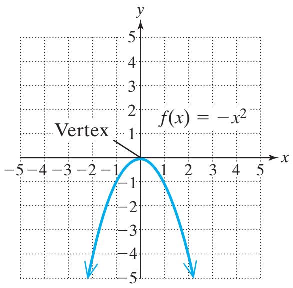


Figure 4-13

Example 2 Identifying Linear, Constant, and Quadratic Functions

Identify each function as linear, constant, quadratic, or none of these.

a. $f(x) = -4$ **b.** $f(x) = x^2 + 3x + 2$

c. $f(x) = 7 - 2x$ **d.** $f(x) = \dfrac{4x + 8}{8}$

Solution:

a. $f(x) = -4$ is a constant function. It is in the form $f(x) = b$, where $b = -4$.

b. $f(x) = x^2 + 3x + 2$ is a quadratic function. It is in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$.

c. $f(x) = 7 - 2x$ is linear. Writing it in the form $f(x) = mx + b$, we get $f(x) = -2x + 7$, where $m = -2$ and $b = 7$.

d. $f(x) = \dfrac{4x + 8}{8}$ is linear. Writing it in the form $f(x) = mx + b$, we get

$$f(x) = \frac{4x}{8} + \frac{8}{8}$$

$$= \frac{1}{2}x + 1, \text{ where } m = \frac{1}{2} \text{ and } b = 1.$$

Skill Practice Identify whether the function is constant, linear, quadratic, or none of these.

3. $m(x) = -2x^2 - 3x + 7$ **4.** $n(x) = -6$

5. $W(x) = \frac{4}{3}x - \frac{1}{2}$ **6.** $R(x) = \frac{4}{3x} - \frac{1}{2}$

4. Finding the *x*- and *y*-Intercepts of a Function Defined by $y = f(x)$

In Section 2.2, we learned that to find an x-intercept, we substitute $y = 0$ and solve the equation for x. Using function notation, this is equivalent to finding the real solutions of the equation $f(x) = 0$. To find a y-intercept, substitute $x = 0$ and solve the equation for y. In function notation, this is equivalent to finding $f(0)$.

Finding the *x*- and *y*-Intercepts of a Function

Given a function defined by $y = f(x)$,

1. The x-intercepts are the real solutions to the equation $f(x) = 0$.
2. The y-intercept is given by $f(0)$.

Example 3 Finding the *x*- and *y*-Intercepts of a Function

Given the function defined by $f(x) = 2x - 4$:

a. Find the x-intercept(s).

b. Find the y-intercept.

c. Graph the function.

Solution:

a. To find the x-intercept(s), find the real solutions to the equation $f(x) = 0$.

$f(x) = 2x - 4$

$0 = 2x - 4$ Substitute $f(x) = 0$.

$4 = 2x$

$2 = x$ The x-intercept is $(2, 0)$.

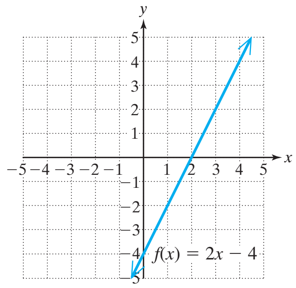


Figure 4-14

b. To find the y-intercept, evaluate $f(0)$.

$f(0) = 2(0) - 4$ Substitute $x = 0$.

$f(0) = -4$ The y-intercept is $(0, -4)$.

c. This function is linear, with a y-intercept of $(0, -4)$, an x-intercept of $(2, 0)$, and a slope of 2 (Figure 4-14).

Skill Practice

7. Consider $f(x) = -5x + 1$.

a. Find the x-intercept.

b. Find the y-intercept.

c. Graph the function.

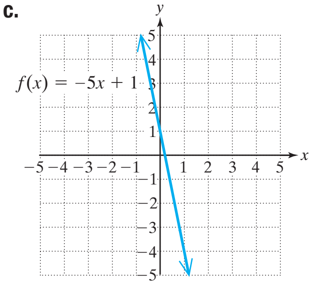
Skill Practice Answers

3. Quadratic **4.** Constant

5. Linear **6.** None of these

7a. $(\frac{1}{5}, 0)$ **b.** $(0, 1)$

c.



Example 4 Finding the x- and y-Intercepts of a Function

For the function pictured in Figure 4-15, estimate

a. The real values of x for which $f(x) = 0$.

b. The value of $f(0)$.

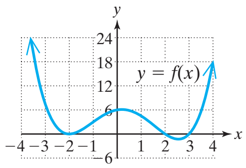


Figure 4-15

Solution:

a. The real values of x for which $f(x) = 0$ are the x-intercepts of the function. For this graph, the x-intercepts are located at $x = -2$, $x = 2$, and $x = 3$.

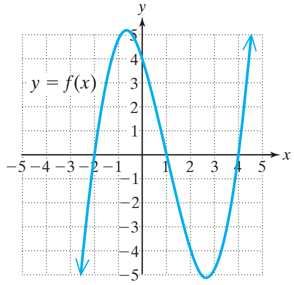
b. The value of $f(0)$ is the value of y at $x = 0$. That is, $f(0)$ is the y-intercept, $f(0) = 6$.

Skill Practice

8. Use the function pictured below.

a. Estimate the real value(s) of x for which $f(x) = 0$

b. Estimate the value of $f(0)$.



5. Determining Intervals of Increasing, Decreasing, or Constant Behavior

The function shown in Figure 4-16 represents monthly household cost for electricity based on average temperature for that month. At lower temperatures, people probably run their heat, and at higher temperatures, people probably run their air-conditioners. This leads to greater energy cost. However, when the average daily temperature is pleasant, such as between 65°F and 70°F, the heater and air-conditioner are turned off and the monthly electric bill is less.

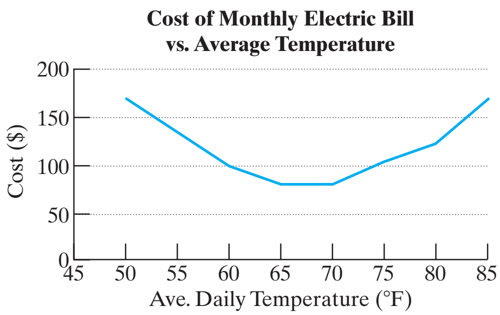


Figure 4-16

- Notice that for temperatures between 50°F and 65°F, the monthly cost decreases. We say that the function is *decreasing* on the interval (50, 65).
- For daily temperatures between 70°F and 85°F, the monthly cost increases. We say that the function is *increasing* on the interval (70, 85).
- For temperatures between 65°F and 70°F, the cost remained the same (or constant). We say that the function is *constant* on the interval (65, 70).

Skill Practice Answers

8a. $x = -2$, $x = 1$, and $x = 4$
b. $f(0) = 4$

In many applications, it is important to note the open intervals where a function is increasing, decreasing, or constant. An open interval, denoted by (a, b), consists of numbers strictly greater than a and strictly less than b.

Intervals Over Which a Function is Increasing, Decreasing, or Constant

Let I be an open interval in the domain of a function, f. Then,

1. f is *increasing* on I if $f(a) < f(b)$ for all $a < b$ on I.
2. f is *decreasing* on I if $f(a) > f(b)$ for all $a < b$ on I.
3. f is *constant* on I if $f(a) = f(b)$ for all a and b on I.

TIP:

- A function is *increasing* on an interval if it goes "uphill" from left to right.
- A function is *decreasing* on an interval if it goes "downhill" from left to right.
- A function is *constant* on an interval if it is "level" or "flat."

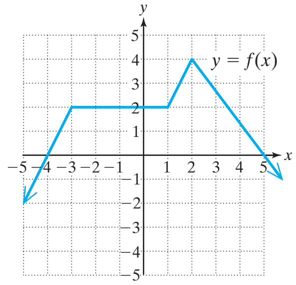
Example 5 Determining Where a Function is Increasing, Decreasing, or Constant

For the function pictured, determine the open interval(s) for which the function is

a. increasing

b. decreasing

c. constant



Solution:

a. As we trace the function from left to right, the y-values increase on the intervals $(-\infty, -3)$ and $(1, 2)$.

b. As we trace the function from left to right, the y-values decrease on the interval $(2, \infty)$.

c. A function is constant on an interval if the y-values remain unchanged (where the graph appears level). This function is constant over the interval $(-3, 1)$.

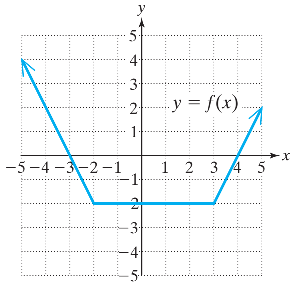
TIP: The intervals over which a function given by $y = f(x)$ is increasing, decreasing, or constant are always expressed in terms of x.

Skill Practice Refer to the function pictured. Find the intervals for which the function is

9a. increasing

b. decreasing

c. constant



Skill Practice Answers

9a. $(3, \infty)$ **b.** $(-\infty, -2)$
c. $(-2, 3)$

Section 4.3 Practice Exercises

Study Skills Exercise

1. Define the key terms.

a. **Linear function** **b.** **Constant function** **c.** **Quadratic function** **d.** **Parabola**

Review Exercises

2. Given: $g = \{(6, 1), (5, 2), (4, 3), (3, 4)\}$

a. Is this relation a function?

b. List the elements in the domain.

c. List the elements in the range.

3. Given: $f = \{(7, 3), (2, 3), (-5, 3)\}$

a. Is this relation a function?

b. List the elements in the domain.

c. List the elements in the range.

4. Given: $f(x) = \sqrt{x + 4}$

a. Evaluate $f(0), f(-3), f(-4)$, and $f(-5)$, if possible.

b. Write the domain of this function in interval notation.

5. Given: $g(x) = \dfrac{2}{x - 3}$

a. Evaluate $g(2), g(4), g(5)$, and $g(3)$, if possible.

b. Write the domain of this function in interval notation.

6. The force (measured in pounds) to stretch a certain spring x inches is given by $f(x) = 3x$. Evaluate $f(3)$ and $f(10)$, and interpret the results in the context of this problem.

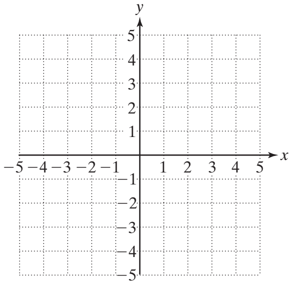
7. The velocity in feet per second of a falling object is given by $V(t) = -32t$, where t is the time in seconds after the object was released. Evaluate $V(2)$ and $V(5)$, and interpret the results in the context of this problem.

Concept 1: Linear and Constant Functions

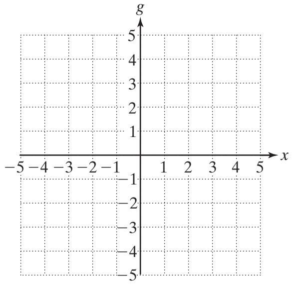
8. Fill in the blank with the word *vertical* or *horizontal*. The graph of a constant function is a ______ line.

9. For the linear function $f(x) = mx + b$, identify the slope and y-intercept.

10. Graph the constant function $f(x) = 2$. Then use the graph to identify the domain and range of f.



11. Graph the linear function $g(x) = -2x + 1$. Then use the graph to identify the domain and range of g.



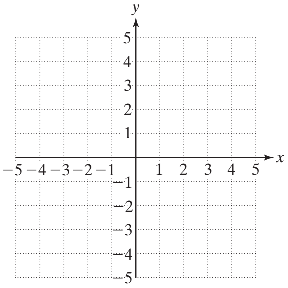
Concept 2: Graphs of Basic Functions

For Exercises 12–17, sketch a graph by completing the table and plotting the points.

12. $f(x) = \dfrac{1}{x}$

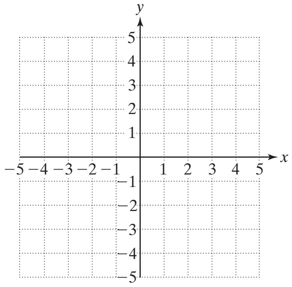
x	$f(x)$
-2	
-1	
$-\frac{1}{2}$	
$-\frac{1}{4}$	

x	$f(x)$
$\frac{1}{4}$	
$\frac{1}{2}$	
1	
2	



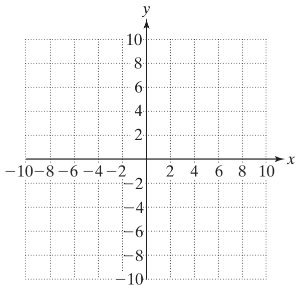
13. $g(x) = |x|$

x	$g(x)$
-2	
-1	
0	
1	
2	



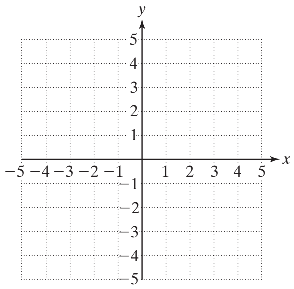
14. $h(x) = x^3$

x	$h(x)$
-2	
-1	
0	
1	
2	



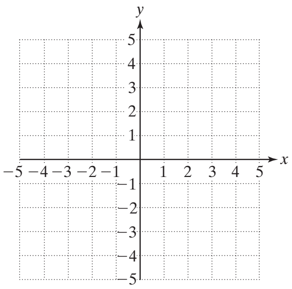
15. $k(x) = x$

x	$k(x)$
-2	
-1	
0	
1	
2	



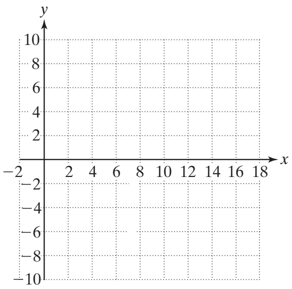
16. $q(x) = x^2$

x	$q(x)$
-2	
-1	
0	
1	
2	



17. $p(x) = \sqrt{x}$

x	$p(x)$
0	
1	
4	
9	
16	



Concept 3: Definition of a Quadratic Function

For Exercises 18–29, determine if the function is constant, linear, quadratic, or none of these.

18. $f(x) = 2x^2 + 3x + 1$ **19.** $g(x) = -x^2 + 4x + 12$ **20.** $k(x) = -3x - 7$ **21.** $h(x) = -x - 3$

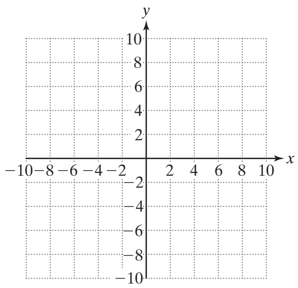
22. $m(x) = \dfrac{4}{3}$ **23.** $n(x) = 0.8$ **24.** $p(x) = \dfrac{2}{3x} + \dfrac{1}{4}$ **25.** $Q(x) = \dfrac{1}{5x} - 3$

26. $t(x) = \dfrac{2}{3}x + \dfrac{1}{4}$ **27.** $r(x) = \dfrac{1}{5}x - 3$ **28.** $w(x) = \sqrt{4 - x}$ **29.** $T(x) = -|x + 10|$

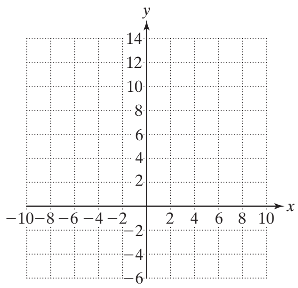
Concept 4: Finding the *x*- and *y*-Intercepts of a Function Defined by $y = f(x)$

For Exercises 30–37, find the x- and y-intercepts, and graph the function.

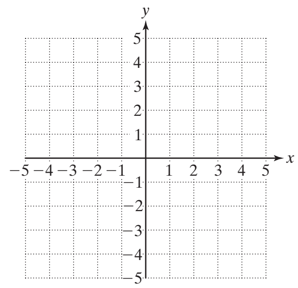
30. $f(x) = 5x - 10$



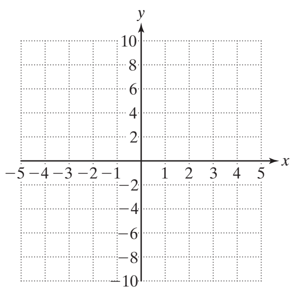
31. $f(x) = -3x + 12$



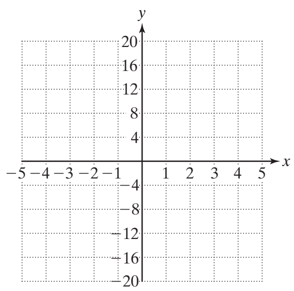
32. $g(x) = -6x + 5$



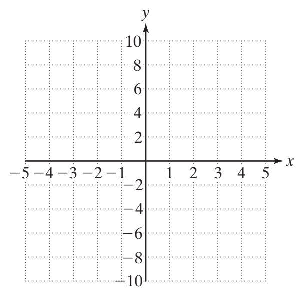
33. $h(x) = 2x + 9$



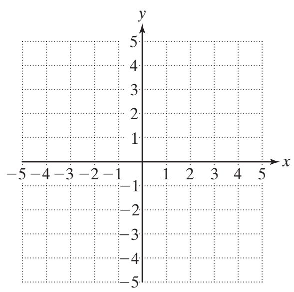
34. $f(x) = 18$



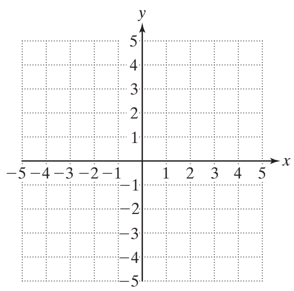
35. $g(x) = -7$



36. $g(x) = \dfrac{2}{3}x + \dfrac{1}{4}$



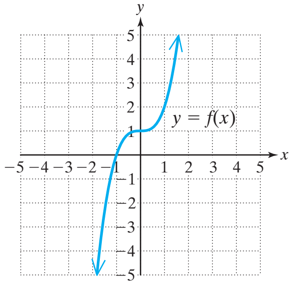
37. $h(x) = -\dfrac{5}{6}x + \dfrac{1}{2}$



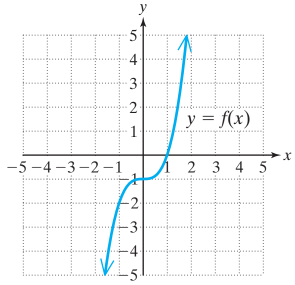
For Exercises 38–43, use the function pictured to estimate

a. The real values of x for which $f(x) = 0$. **b.** The value of $f(0)$.

38.



39.



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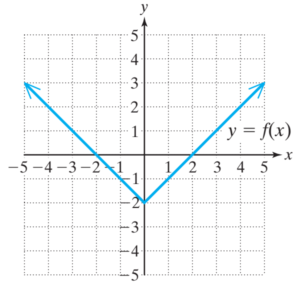

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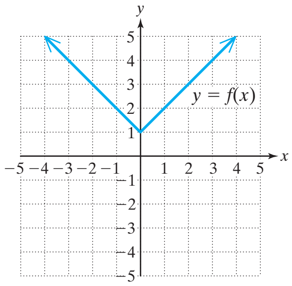

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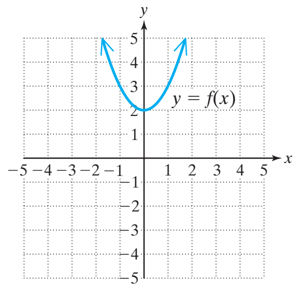
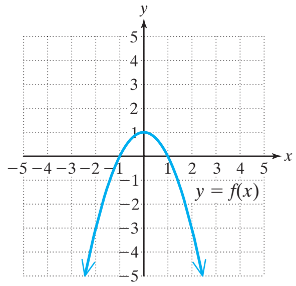


43.



For Exercises 44–53,

a. Identify the domain of the function.

b. Identify the y-intercept of the function.

c. Match the function with its graph by recognizing the basic shape of the function and using the results from parts (a) and (b). Plot additional points if necessary.

44. $q(x) = 2x^2$

45. $p(x) = -2x^2 + 1$

46. $h(x) = x^3 + 1$

47. $k(x) = x^3 - 2$

48. $r(x) = \sqrt{x + 1}$

49. $s(x) = \sqrt{x + 4}$

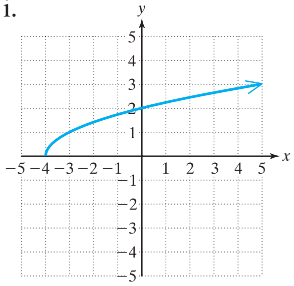
50. $f(x) = \dfrac{1}{x - 3}$

51. $g(x) = \dfrac{1}{x + 1}$

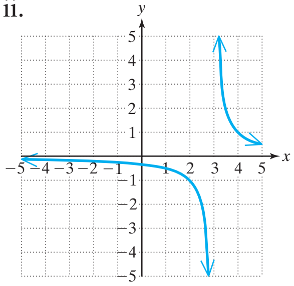
52. $k(x) = |x + 2|$

53. $h(x) = |x - 1| + 2$

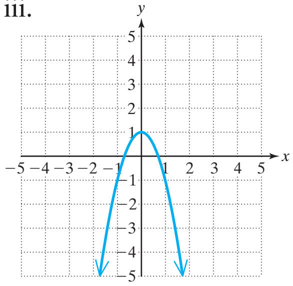
i.



ii.



iii.



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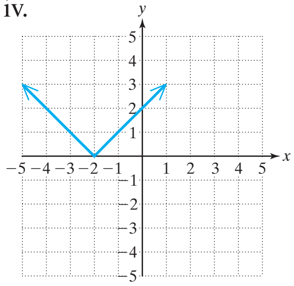
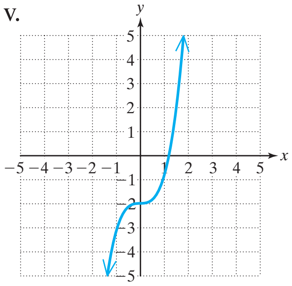
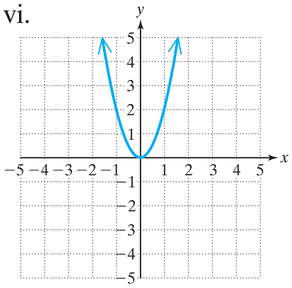


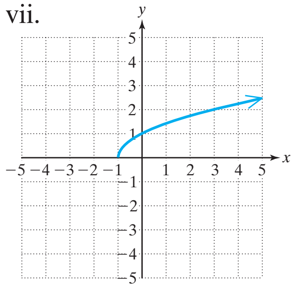
v.



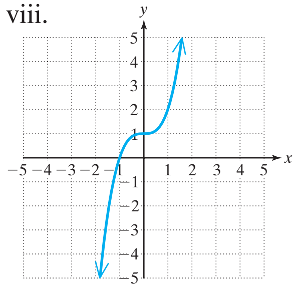
vi.



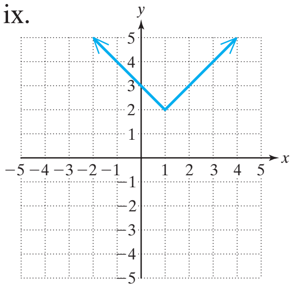
vii.



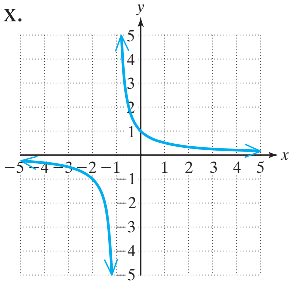
viii.



ix.



x.

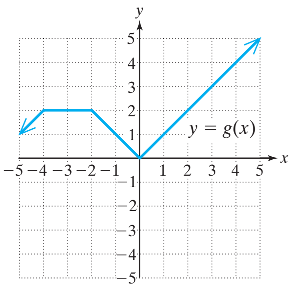


Concept 5: Determining Intervals of Increasing, Decreasing, or Constant Behavior

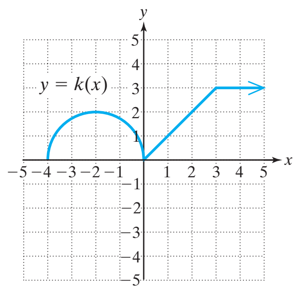
For Exercises 54–57, give the open interval(s) over which the function is

a. increasing **b.** decreasing **c.** constant

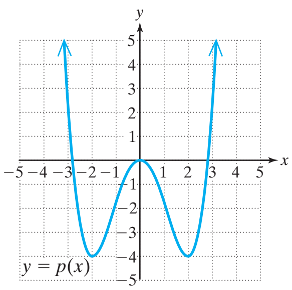
54.



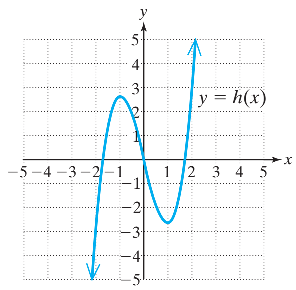
55.



56.



57.



For Exercises 58–63, refer to the graphs of the six basic functions (see page 280). For each function, give the open interval(s) over which the function is

a. increasing **b.** decreasing **c.** constant

58. $f(x) = x$

59. $g(x) = x^2$

60. $h(x) = x^3$

61. $m(x) = |x|$

62. $n(x) = \sqrt{x}$

63. $p(x) = \dfrac{1}{x}$

64. Refer back to the graph in Exercise 54. The function is increasing on the interval $(0, \infty)$. Suppose we arbitrarily select two values of x on this interval such as $x = 1$ and $x = 3$. Is it true that $g(1) < g(3)$? How does this relate to the definition of a function increasing on an interval?

65. Refer back to the graph in Exercise 55. The function is decreasing on the interval $(-2, 0)$. Suppose we arbitrarily select two values of x on this interval, such as $x = -1.5$ and $x = -0.5$. Is it true that $k(-1.5) > k(-0.5)$? How does this relate to the definition of a function decreasing on an interval?

Graphing Calculator Exercises

For Exercises 66–71, use a graphing calculator to graph the basic functions. Verify your answers from the table on page 280.

66. $f(x) = x$

67. $f(x) = x^2$

68. $f(x) = x^3$

69. $f(x) = |x|$

70. $f(x) = \sqrt{x}$

71. $f(x) = \dfrac{1}{x}$

Section 4.4 Variation

Concepts

1. Definition of Direct and Inverse Variation
2. Translations Involving Variation
3. Applications of Variation

1. Definition of Direct and Inverse Variation

In this section, we introduce the concept of variation. Direct and inverse variation models can show how one quantity varies in proportion to another.

Definition of Direct and Inverse Variation

Let k be a nonzero constant real number. Then the following statements are equivalent:

1. y varies **directly** as x.
y is directly proportional to x.
$\Big\}\quad y = kx$

2. y varies **inversely** as x.
y is inversely proportional to x.
$\Big\}\quad y = \dfrac{k}{x}$

Note: The value of k is called the constant of variation.

For a car traveling 30 mph, the equation $d = 30t$ indicates that the distance traveled is *directly proportional* to the time of travel. For positive values of k, when two variables are directly related, as one variable increases, the other variable will also increase. Likewise, if one variable decreases, the other will decrease. In the equation $d = 30t$, the longer the time of the trip, the greater the distance traveled. The shorter the time of the trip, the shorter the distance traveled.

For positive values of k, when two variables are *inversely related*, as one variable increases, the other will decrease, and vice versa. Consider a car traveling between Toronto and Montreal, a distance of 500 km. The time required to make the trip is inversely proportional to the speed of travel: $t = 500/r$. As the rate of speed r increases, the quotient $500/r$ will decrease. Hence, the time will decrease. Similarly, as the rate of speed decreases, the trip will take longer.

2. Translations Involving Variation

The first step in using a variation model is to translate an English phrase into an equivalent mathematical equation.

Example 1 Translating to a Variation Model

Translate each expression into an equivalent mathematical model.

a. The circumference of a circle varies directly as the radius.

b. At a constant temperature, the volume of a gas varies inversely as the pressure.

c. The length of time of a meeting is directly proportional to the *square* of the number of people present.

Solution:

a. Let C represent circumference and r represent radius. The variables are directly related, so use the model $C = kr$.

b. Let V represent volume and P represent pressure. Because the variables are inversely related, use the model $V = \frac{k}{P}$.

c. Let t represent time and let N be the number of people present at a meeting. Because t is directly related to N^2, use the model $t = kN^2$.

Skill Practice Translate to a variation model.

1. The time t it takes to drive a particular distance is inversely proportional to the speed s.

2. The amount of your paycheck P varies directly with the number of hours h that you work.

3. q varies inversely as the square of t.

Sometimes a variable varies directly as the product of two or more other variables. In this case, we have joint variation.

Definition of Joint Variation

Let k be a nonzero constant real number. Then the following statements are equivalent:

$$\left.\begin{array}{l} y \text{ varies } \textbf{jointly} \text{ as } w \text{ and } z. \\ y \text{ is jointly proportional to } w \text{ and } z. \end{array}\right\} \quad y = kwz$$

Skill Practice Answers

1. $t = \frac{k}{s}$ **2.** $P = kh$

3. $q = \frac{k}{t^2}$

Example 2 Translating to a Variation Model

Translate each expression into an equivalent mathematical model.

a. y varies jointly as u and the square root of v.

b. The gravitational force of attraction between two planets varies jointly as the product of their masses and inversely as the square of the distance between them.

Solution:

a. $y = ku\sqrt{v}$

b. Let m_1 and m_2 represent the masses of the two planets. Let F represent the gravitational force of attraction and d represent the distance between the planets. The variation model is $F = \dfrac{km_1m_2}{d^2}$.

Skill Practice Translate to a variation model.

4. a varies jointly as b and c.

5. x varies directly as the square root of y and inversely as z.

3. Applications of Variation

Consider the variation models $y = kx$ and $y = k/x$. In either case, if values for x and y are known, we can solve for k. Once k is known, we can use the variation equation to find y if x is known, or to find x if y is known. This concept is the basis for solving many problems involving variation.

Steps to Find a Variation Model

1. Write a general variation model that relates the variables given in the problem. Let k represent the constant of variation.
2. Solve for k by substituting known values of the variables into the model from step 1.
3. Substitute the value of k into the original variation model from step 1.

Example 3 Solving an Application Involving Direct Variation

The variable z varies directly as w. When w is 16, z is 56.

a. Write a variation model for this situation. Use k as the constant of variation.

b. Solve for the constant of variation.

c. Find the value of z when w is 84.

Solution:

a. $z = kw$

Skill Practice Answers

4. $a = kbc$ **5.** $x = \dfrac{k\sqrt{y}}{z}$

b. $z = kw$

$56 = k(16)$ Substitute known values for z and w. Then solve for the unknown value of k.

$\frac{56}{16} = \frac{k(16)}{16}$ To isolate k, divide both sides by 16.

$\frac{7}{2} = k$ Simplify $\frac{56}{16}$ to $\frac{7}{2}$.

c. With the value of k known, the variation model can now be written as $z = \frac{7}{2}w$.

$z = \frac{7}{2}(84)$ To find z when $w = 84$, substitute $w = 84$ into the equation.

$z = 294$

Skill Practice

6. The variable q varies directly as the square of v. When v is 2, q is 40.
   a. Write a variation model for this relationship.
   b. Solve for the constant of variation.
   c. Find q when $v = 7$.

Example 4 Solving an Application Involving Direct Variation

The speed of a racing canoe in still water varies directly as the square root of the length of the canoe.

a. If a 16-ft canoe can travel 6.2 mph in still water, find a variation model that relates the speed of a canoe to its length.

b. Find the speed of a 25-ft canoe.

Solution:

a. Let s represent the speed of the canoe and L represent the length. The general variation model is $s = k\sqrt{L}$. To solve for k, substitute the known values for s and L.

$s = k\sqrt{L}$

$6.2 = k\sqrt{16}$ Substitute $s = 6.2$ mph and $L = 16$ ft.

$6.2 = k \cdot 4$

$\frac{6.2}{4} = \frac{\cancel{4}k}{\cancel{4}}$ Solve for k.

$k = 1.55$

$s = 1.55\sqrt{L}$ Substitute $k = 1.55$ into the model $s = k\sqrt{L}$.

b. $s = 1.55\sqrt{L}$

$= 1.55\sqrt{25}$ Find the speed when $L = 25$ ft.

$= 7.75$ mph

Skill Practice Answers

6a. $q = kv^2$ **b.** $k = 10$
c. $q = 490$

Skill Practice

7. The amount of water needed by a mountain hiker varies directly as the time spent hiking. The hiker needs 2.4 L for a 3-hr hike.
   a. Write a model that relates the amount of water needed to the time of the hike.
   b. How much water will be needed for a 5-hr hike?

Example 5 Solving an Application Involving Inverse Variation

The loudness of sound measured in decibels (dB) varies inversely as the square of the distance between the listener and the source of the sound. If the loudness of sound is 17.92 dB at a distance of 10 ft from a stereo speaker, what is the decibel level 20 ft from the speaker?

Solution:

Let L represent the loudness of sound in decibels and d represent the distance in feet. The inverse relationship between decibel level and the square of the distance is modeled by

$$L = \frac{k}{d^2}$$

$$17.92 = \frac{k}{(10)^2} \qquad \text{Substitute } L = 17.92 \text{ dB and } d = 10 \text{ ft.}$$

$$17.92 = \frac{k}{100}$$

$$(17.92)100 = \frac{k}{\cancel{100}} \cdot \cancel{100} \qquad \text{Solve for } k \text{ (clear fractions).}$$

$$k = 1792$$

$$L = \frac{1792}{d^2} \qquad \text{Substitute } k = 1792 \text{ into the original model } L = \frac{k}{d^2}.$$

With the value of k known, we can find L for any value of d.

$$L = \frac{1792}{(20)^2} \qquad \text{Find the loudness when } d = 20 \text{ ft.}$$

$$= 4.48 \text{ dB}$$

Notice that the loudness of sound is 17.92 dB at a distance 10 ft from the speaker. When the distance from the speaker is increased to 20 ft, the decibel level decreases to 4.48 dB. This is consistent with an inverse relationship. For $k > 0$, as one variable is increased, the other is decreased. It also seems reasonable that the farther one moves away from the source of a sound, the softer the sound becomes.

Skill Practice

8. The yield on a bond varies inversely as the price. The yield on a particular bond is 4% when the price is \$100. Find the yield when the price is \$80.

Skill Practice Answers

7a. $w = 0.8t$ b. 4 L 8. 5%

Example 6 Solving an Application Involving Joint Variation

In the early morning hours of August 29, 2005, Hurricane Katrina plowed into the Gulf Coast of the United States, bringing unprecedented destruction to southern Louisiana, Mississippi, and Alabama.

The kinetic energy of an object varies jointly as the weight of the object at sea level and as the square of its velocity. During a hurricane, a $\frac{1}{2}$-lb stone traveling at 60 mph has 81 joules (J) of kinetic energy. Suppose the wind speed doubles to 120 mph. Find the kinetic energy.

Solution:

Let E represent the kinetic energy, let w represent the weight, and let v represent the velocity of the stone. The variation model is

$$E = kwv^2$$

$$81 = k(0.5)(60)^2 \qquad \text{Substitute } E = 81 \text{ J}, w = 0.5 \text{ lb, and } v = 60 \text{ mph.}$$

$$81 = k(0.5)(3600) \qquad \text{Simplify exponents.}$$

$$81 = k(1800)$$

$$\frac{81}{1800} = \frac{k(\cancel{1800})}{\cancel{1800}} \qquad \text{Divide by 1800.}$$

$$0.045 = k \qquad \text{Solve for } k.$$

With the value of k known, the model $E = kwv^2$ can be written as $E = 0.045wv^2$. We now find the kinetic energy of a $\frac{1}{2}$-lb stone traveling at 120 mph.

$$E = 0.045(0.5)(120)^2$$
$$= 324$$

The kinetic energy of a $\frac{1}{2}$-lb stone traveling at 120 mph is 324 J.

Skill Practice

9. The amount of simple interest earned in an account varies jointly as the interest rate and time of the investment. An account earns \$40 in 2 years at 4% interest. How much interest would be earned in 3 years at a rate of 5%?

In Example 6, when the velocity increased by 2 times, the kinetic energy increased by 4 times (note that $324 \text{ J} = 4 \cdot 81 \text{ J}$). This factor of 4 occurs because the kinetic energy is proportional to the *square* of the velocity. When the velocity increased by 2 times, the kinetic energy increased by 2^2 times.

Skill Practice Answers

9. \$75

Section 4.4 Practice Exercises

Study Skills Exercises

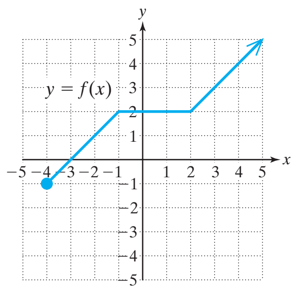
1. It is not too early to think about your final exam. Write the page number of the cumulative review for Chapters 1–4. Make this exercise set part of your homework this week.

2. Define the key terms.

a. Direct variation **b. Inverse variation** **c. Joint variation**

Review Exercises

For Exercises 3–8, refer to the graph.



3. Find $f(-3)$.

4. Find $f(0)$.

5. Find the value(s) of x for which $f(x) = 1$.

6. Find the value(s) of x for which $f(x) = 2$.

7. Write the domain of f.

8. Write the range of f.

Concept 1: Definition of Direct and Inverse Variation

9. Suppose y varies directly as x, and $k > 0$.

a. If x increases, then will y increase or decrease?

b. If x decreases, will y increase or decrease?

10. Suppose y varies inversely as x, and $k > 0$.

a. If x increases, then will y increase or decrease?

b. If x decreases, then will y increase or decrease?

Concept 2: Translations Involving Variation

For Exercises 11–18, write a variation model. Use k as the constant of variation.

11. T varies directly as q.

12. P varies inversely as r.

13. W varies inversely as the square of p.

14. Y varies directly as the square root of z.

15. Q is directly proportional to x and inversely proportional to the cube of y.

16. M is directly proportional to the square of p and inversely proportional to the cube of n.

17. L varies jointly as w and the square root of v.

18. X varies jointly as w and the square of y.

Concept 3: Applications of Variation

For Exercises 19–24, find the constant of variation k.

19. y varies directly as x, and when x is 4, y is 18.

20. m varies directly as x and when x is 8, m is 22.

21. p is inversely proportional to q and when q is 16, p is 32.

22. T is inversely proportional to x and when x is 40, T is 200.

23. y varies jointly as w and v. When w is 50 and v is 0.1, y is 8.75.

24. N varies jointly as t and p. When t is 1 and p is 7.5, N is 330.

Solve Exercises 25–30 by using the steps found on page 292.

25. Z varies directly as the square of w, and $Z = 14$ when $w = 4$. Find Z when $w = 8$.

26. Q varies inversely as the square of p, and $Q = 4$ when $p = 3$. Find Q when $p = 2$.

27. L varies jointly as a and the square root of b, and $L = 72$ when $a = 8$ and $b = 9$. Find L when $a = \frac{1}{2}$ and $b = 36$.

28. Y varies jointly as the cube of x and the square root of w, and $Y = 128$ when $x = 2$ and $w = 16$. Find Y when $x = \frac{1}{2}$ and $w = 64$.

29. B varies directly as m and inversely as n, and $B = 20$ when $m = 10$ and $n = 3$. Find B when $m = 15$ and $n = 12$.

30. R varies directly as s and inversely as t, and $R = 14$ when $s = 2$ and $t = 9$. Find R when $s = 4$ and $t = 3$.

For Exercises 31–42, use a variation model to solve for the unknown value.

31. The amount of pollution entering the atmosphere varies directly as the number of people living in an area. If 80,000 people cause 56,800 tons of pollutants, how many tons enter the atmosphere in a city with a population of 500,000?

32. The area of a picture projected on a wall varies directly as the square of the distance from the projector to the wall. If a 10-ft distance produces a 16-ft^2 picture, what is the area of a picture produced when the projection unit is moved to a distance 20 ft from the wall?

33. The stopping distance of a car is directly proportional to the square of the speed of the car. If a car traveling at 40 mph has a stopping distance of 109 ft, find the stopping distance of a car that is traveling at 25 mph. (Round your answer to 1 decimal place.)

34. The intensity of a light source varies inversely as the square of the distance from the source. If the intensity is 48 lumens (lm) at a distance of 5 ft, what is the intensity when the distance is 8 ft?

35. The current in a wire varies directly as the voltage and inversely as the resistance. If the current is 9 amperes (A) when the voltage is 90 volts (V) and the resistance is 10 ohms (Ω), find the current when the voltage is 185 V and the resistance is 10 Ω.

36. The power in an electric circuit varies jointly as the current and the square of the resistance. If the power is 144 watts (W) when the current is 4 A and the resistance is 6 Ω, find the power when the current is 3 A and the resistance is 10 Ω.

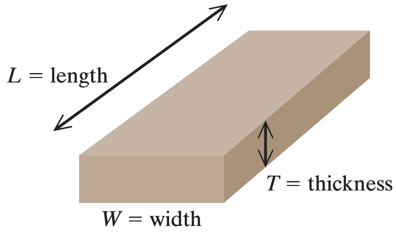
37. The resistance of a wire varies directly as its length and inversely as the square of its diameter. A 40-ft wire with 0.1-in. diameter has a resistance of 4 Ω. What is the resistance of a 50-ft wire with a diameter of 0.20 in.?

38. The frequency of a vibrating string is inversely proportional to its length. A 24-in. piano string vibrates at 252 cycles/sec. What is the frequency of an 18-in. piano string?

39. The weight of a medicine ball varies directly as the cube of its radius. A ball with a radius of 3 in. weighs 4.32 lb. How much would a medicine ball weigh if its radius were 5 in.?

40. The surface area of a cube varies directly as the square of the length of an edge. The surface area is 24 ft^2 when the length of an edge is 2 ft. Find the surface area of a cube with an edge that is 5 ft.

41. The strength of a wooden beam varies jointly as the width of the beam and the square of the thickness of the beam and inversely as the length of the beam. A beam that is 48 in. long, 6 in. wide, and 2 in. thick can support a load of 417 lb. Find the maximum load that can be safely supported by a board that is 12 in. wide, 72 in. long, and 4 in. thick.



42. The period of a pendulum is the length of time required to complete one swing back and forth. The period varies directly as the square root of the length of the pendulum. If it takes 1.8 sec for a 0.81-m pendulum to complete one period, what is the period of a 1-m pendulum?

Expanding Your Skills

43. The area A of a square varies directly as the square of the length l of its sides.

a. Write a general variation model with k as the constant of variation.

b. If the length of the sides is doubled, what effect will that have on the area?

c. If the length of the sides is tripled, what effect will that have on the area?

44. In a physics laboratory, a spring is fixed to the ceiling. With no weight attached to the end of the spring, the spring is said to be in its equilibrium position. As weights are applied to the end of the spring, the force stretches the spring a distance d from its equilibrium position. A student in the laboratory collects the following data:

Force F (lb)	2	4	6	8	10
Distance d (cm)	2.5	5.0	7.5	10.0	12.5

a. Based on the data, do you suspect a direct relationship between force and distance or an inverse relationship?

b. Find a variation model that describes the relationship between force and distance.

Chapter 4 SUMMARY

Section 4.1 Introduction to Relations

Key Concepts

Any set of ordered pairs (x, y) is called a **relation in *x* and *y***.

The **domain** of a relation is the set of first components in the ordered pairs in the relation. The **range** of a relation is the set of second components in the ordered pairs.

Examples

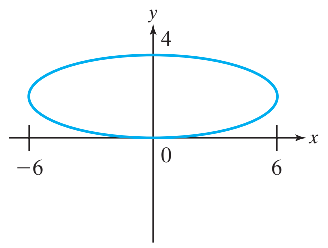
Example 1

Let $A = \{(0, 0), (1, 1), (2, 4), (3, 9), (-1, 1), (-2, 4), (-3, 9)\}$.

Domain of A: $\{0, 1, 2, 3, -1, -2, -3\}$

Range of A: $\{0, 1, 4, 9\}$

Example 2



Domain: $[-6, 6]$

Range: $[0, 4]$

Section 4.2 Introduction to Functions

Key Concepts

Given a relation in x and y, we say "***y* is a function of *x***" if for every element x in the domain, there corresponds exactly one element y in the range.

The Vertical Line Test for Functions

Consider a relation defined by a set of points (x, y) in a rectangular coordinate system. Then the graph defines y as a function of x if no vertical line intersects the graph in more than one point.

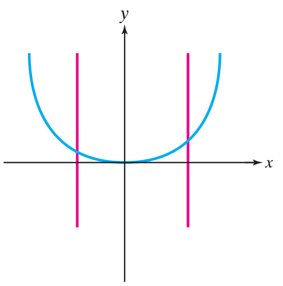
Examples

Example 1

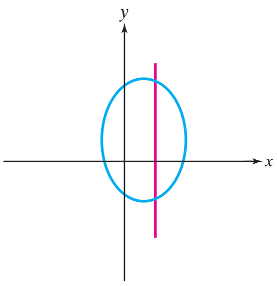
Function $\{(1, 3), (2, 5), (6, 3)\}$

Nonfunction $\{(1, 3), (2, 5), (1, 4)\}$

Example 2



Function



Not a Function

Function Notation

$f(x)$ is the value of the function f at x.

Example 3

Given $f(x) = -3x^2 + 5x$, find $f(-2)$.

$$f(-2) = -3(-2)^2 + 5(-2)$$
$$= -12 - 10$$
$$= -22$$

The domain of a function defined by $y = f(x)$ is the set of x-values that when substituted into the function produces a real number. In particular,

- Exclude values of x that make the denominator of a fraction zero.
- Exclude values of x that make a negative value within a square root.

Example 4

Find the domain.

1. $f(x) = \dfrac{x+4}{x-5}$; $(-\infty, 5) \cup (5, \infty)$
2. $f(x) = \sqrt{x-3}$; $[3, \infty)$
3. $f(x) = 3x^2 - 5$; $(-\infty, \infty)$

Section 4.3 Graphs of Functions

Key Concepts

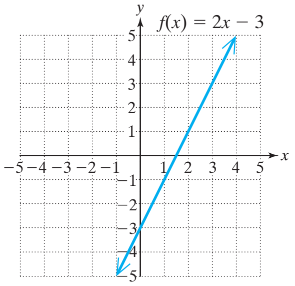
A function of the form $f(x) = mx + b$ $(m \neq 0)$ is a **linear function**. Its graph is a line with slope m and y-intercept $(0, b)$.

A function of the form $f(x) = k$ is a **constant function**. Its graph is a horizontal line.

Examples

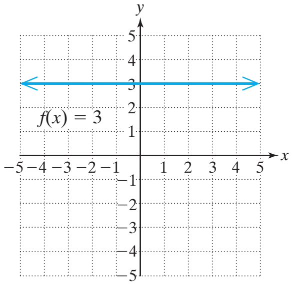
Example 1

$f(x) = 2x - 3$



Example 2

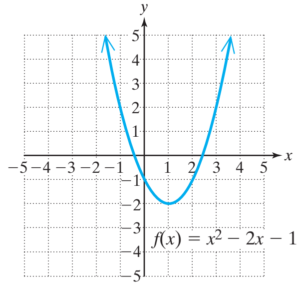
$f(x) = 3$



A function of the form $f(x) = ax^2 + bx + c$ $(a \neq 0)$ is a **quadratic function**. Its graph is a **parabola**.

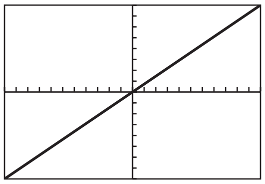
Example 3

$f(x) = x^2 - 2x - 1$

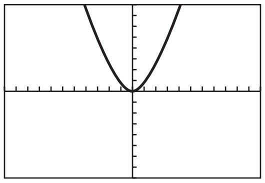


Graphs of basic functions:

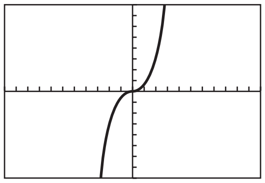
$f(x) = x$



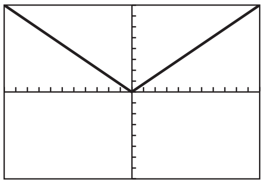
$f(x) = x^2$



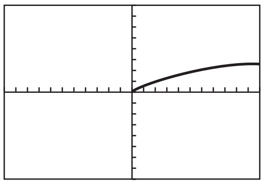
$f(x) = x^3$



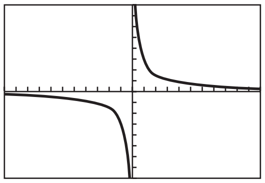
$f(x) = |x|$



$f(x) = \sqrt{x}$



$f(x) = \dfrac{1}{x}$

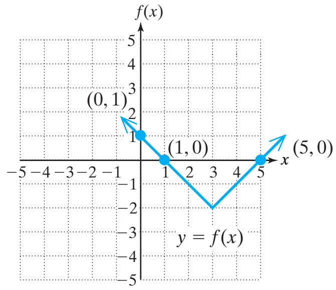


The x-intercepts of a function are determined by finding the real solutions to the equation $f(x) = 0$.

The y-intercept of a function is at $f(0)$.

Example 4

Find the x- and y-intercepts for the function pictured.



$f(x) = 0$, when $x = 1$ and $x = 5$.

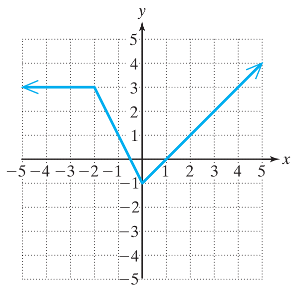
The x-intercepts are $(1, 0)$ and $(5, 0)$.

$f(0) = 1$. The y-intercept is $(0, 1)$.

Let I be an open interval in the domain of a function, f. Then,

1. f is *increasing* on I if $f(a) < f(b)$ for all $a < b$ on I.
2. f is *decreasing* on I if $f(a) > f(b)$ for all $a < b$ on I.
3. f is *constant* on I if $f(a) = f(b)$ for all a and b on I.

Example 5



f is increasing on $(0, \infty)$

f is decreasing on $(-2, 0)$

f is constant on $(-\infty, -2)$

Section 4.4 Variation

Key Concepts

Direct Variation

y varies directly as x.
y is directly proportional to x. } $y = kx$

Inverse Variation

y varies inversely as x.
y is inversely proportional to x. } $y = \dfrac{k}{x}$

Joint Variation

y varies jointly as w and z.
y is jointly proportional to w and z. } $y = kwz$

Steps to Find a Variation Model

1. Write a general variation model that relates the variables given in the problem. Let k represent the constant of variation.
2. Solve for k by substituting known values of the variables into the model from step 1.
3. Substitute the value of k into the original variation model from step 1.

Examples

Example 1

t varies directly as the square root of x.

$t = k\sqrt{x}$

Example 2

W is inversely proportional to the cube of x.

$W = \dfrac{k}{x^3}$

Example 3

y is jointly proportional to x and to the square of z.

$y = kxz^2$

Example 4

C varies directly as the square root of d and inversely as t. If $C = 12$ when d is 9 and t is 6, find C if d is 16 and t is 12.

Step 1: $C = \dfrac{k\sqrt{d}}{t}$

Step 2: $12 = \dfrac{k\sqrt{9}}{6} \Rightarrow 12 = \dfrac{k \cdot 3}{6} \Rightarrow k = 24$

Step 3: $C = \dfrac{24\sqrt{d}}{t} \Rightarrow C = \dfrac{24\sqrt{16}}{12} \Rightarrow C = 8$

Chapter 4 Review Exercises

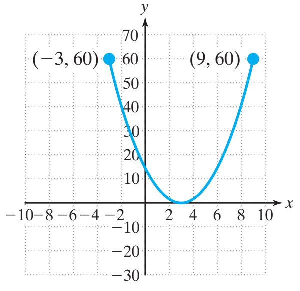
Section 4.1

1. Write a relation with four ordered pairs for which the first element is the name of a parent and the second element is the name of the parent's child.

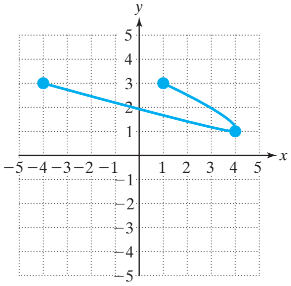
For Exercises 2–5, find the domain and range.

2. $\left\{\left(\frac{1}{3}, 10\right), \left(6, -\frac{1}{2}\right), \left(\frac{1}{4}, 4\right), \left(7, \frac{2}{5}\right)\right\}$

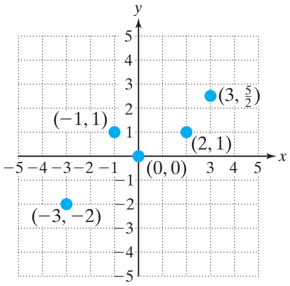
3.



4.

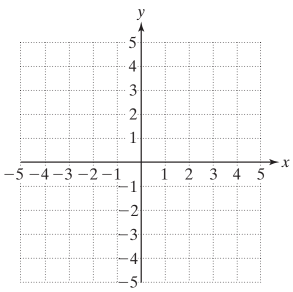


5.

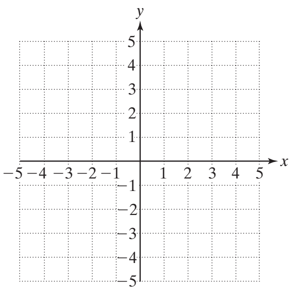


Section 4.2

6. Sketch a relation that is *not* a function. (Answers may vary.)



7. Sketch a relation that *is* a function. (Answers may vary.)



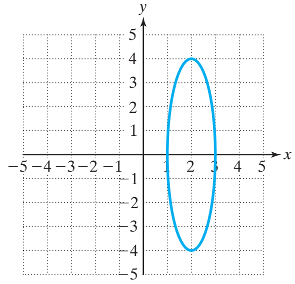
For Exercises 8–13:

a. Determine whether the relation defines y as a function of x.

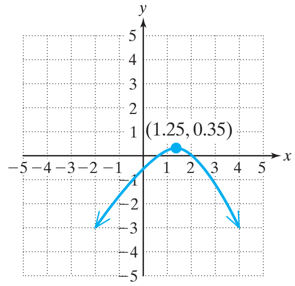
b. Find the domain.

c. Find the range.

8.



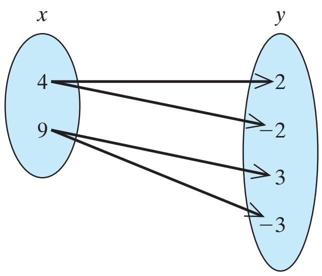
9.



10. $\{(1, 3), (2, 3), (3, 3), (4, 3)\}$

11. $\{(0, 2), (0, 3), (4, 4), (0, 5)\}$

12.



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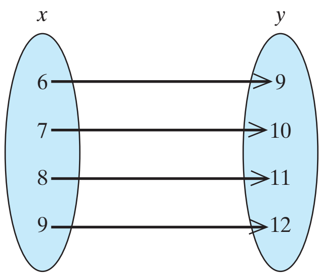


For Exercises 14–21, find the function values given $f(x) = 6x^2 - 4$.

14. $f(0)$

15. $f(1)$

16. $f(-1)$

17. $f(t)$

18. $f(b)$

19. $f(\pi)$

20. $f(\square)$

21. $f(-2)$

For Exercises 22–25, write the domain of each function in interval notation.

22. $g(x) = 7x^3 + 1$

23. $h(x) = \dfrac{x + 10}{x - 11}$

24. $k(x) = \sqrt{x - 8}$

25. $w(x) = \sqrt{x + 2}$

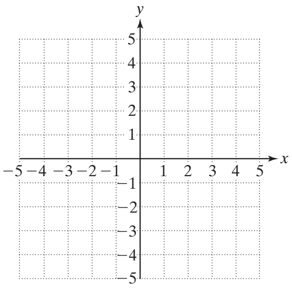
26. Anita is a waitress and makes \$6 per hour plus tips. Her tips average \$5 per table. In one 8-hr shift, Anita's pay can be described by $p(x) = 48 + 5x$, where x represents the number of tables she waits on. Find out how much Anita will earn if she waits on

a. 10 tables **b.** 15 tables **c.** 20 tables

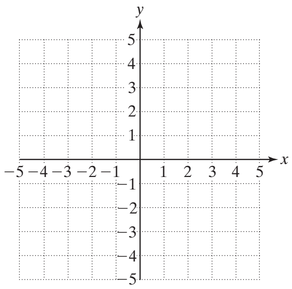
Section 4.3

For Exercises 27–32, sketch the functions from memory.

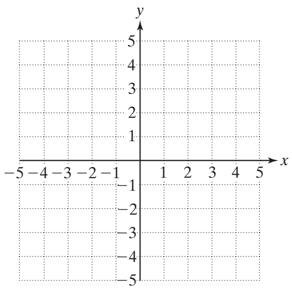
27. $h(x) = x$



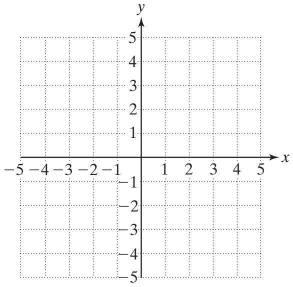
28. $f(x) = x^2$



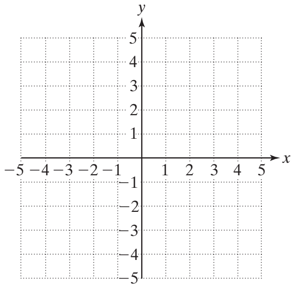
29. $g(x) = x^3$



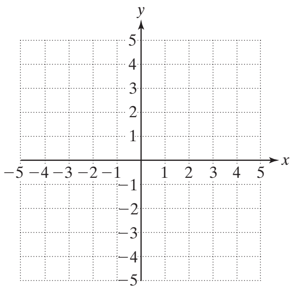
30. $w(x) = |x|$



31. $s(x) = \sqrt{x}$

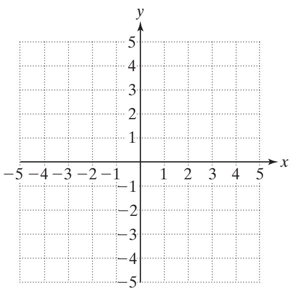


32. $r(x) = \dfrac{1}{x}$

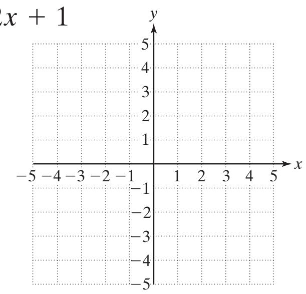


For Exercises 33–34, sketch the function and determine the open intervals for which the function is increasing, decreasing, and constant.

33. $q(x) = 3$



34. $k(x) = 2x + 1$



For Exercises 35–36, find the x- and y-intercepts.

35. $p(x) = 4x - 7$

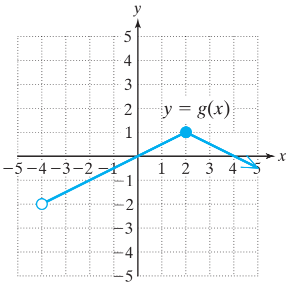
36. $q(x) = -2x + 9$

37. The function defined by $b(t) = 0.7t + 4.5$ represents the per capita consumption of bottled water in the United States between 1985 and 2005. The values of $b(t)$ are measured in gallons, and $t = 0$ corresponds to the year 1985. (Source: U.S. Department of Agriculture.)

a. Evaluate $b(0)$ and $b(7)$ and interpret the results in the context of this problem.

b. What is the slope of this function? Interpret the slope in the context of this problem.

For Exercises 38–45, refer to the graph.



38. Find $g(-2)$.

39. Find $g(2)$.

40. For what value(s) of x is $g(x) = 0$?

41. For what value(s) of x is $g(x) = -4$?

42. Write the domain of g.

43. Write the range of g.

44. For what open interval(s) is g increasing?

45. For what open interval(s) is g decreasing?

46. Given: $r(x) = 2\sqrt{x - 4}$

a. Find $r(4)$, $r(5)$, and $r(8)$.

b. What is the domain of r?

47. Given: $h(x) = \dfrac{3}{x - 3}$

a. Find $h(-3)$, $h(-1)$, $h(0)$, $h(2)$, $h(3)$, and $h(4)$.

b. What is the domain of h?

Section 4.4

48. The force applied to a spring varies directly with the distance that the spring is stretched. When 6 lb of force is applied, the spring stretches 2 ft.

a. Write a variation model using k as the constant of variation.

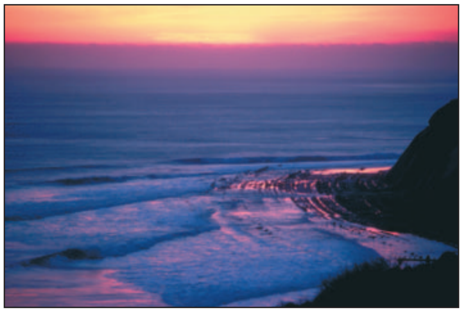
b. Find k.

c. How many feet will the spring stretch when 5 lb of pressure is applied?

49. Suppose y varies directly with the cube of x and $y = 32$ when $x = 2$. Find y when $x = 4$.

50. Suppose y varies jointly with x and the square root of z, and $y = 3$ when $x = 3$ and $z = 4$. Find y when $x = 8$ and $z = 9$.

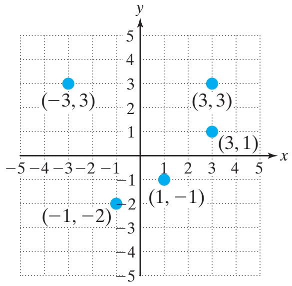
51. The distance d that one can see to the horizon varies directly as the square root of the height above sea level. If a person 25 m above sea level can see 30 km, how far can a person see if she is 64 m above sea level?



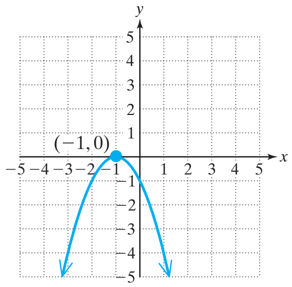
Chapter 4 Test

For Exercises 1–3, **a.** determine if the relation defines y as a function of x, **b.** identify the domain, and **c.** identify the range.

1.



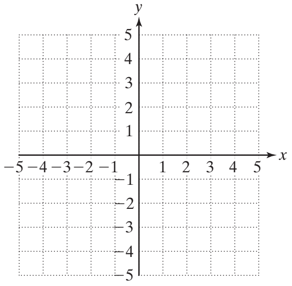
2.



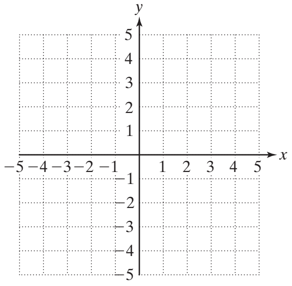
3. Explain how to find the x- and y-intercepts of a function defined by $y = f(x)$.

Graph the functions.

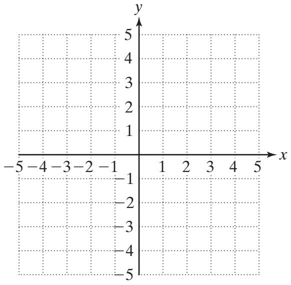
4. $f(x) = -3x - 1$



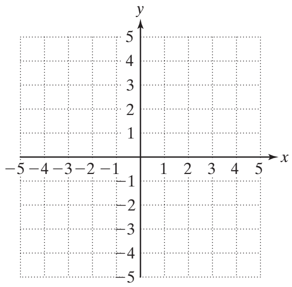
5. $k(x) = -2$



6. $p(x) = x^2$



7. $w(x) = |x|$



For Exercises 8–10, write the domain in interval notation.

8. $f(x) = \dfrac{x - 5}{x + 7}$

9. $f(x) = \sqrt{x + 7}$

10. $h(x) = (x + 7)(x - 5)$

11. Given: $r(x) = x^2 - 2x + 1$

a. Find $r(-2)$, $r(-1)$, $r(0)$, $r(2)$, and $r(3)$.

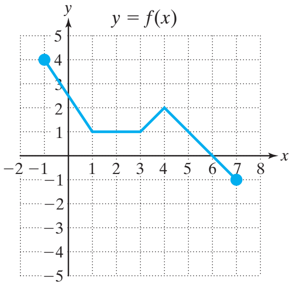
b. What is the domain of r?

12. The function defined by $s(t) = 1.6t + 36$ approximates the per capita consumption of soft drinks in the United States between 1985 and 2006. The values of $s(t)$ are measured in gallons, and $t = 0$ corresponds to the year 1985. (Source: U.S. Department of Agriculture.)

a. Evaluate $s(0)$ and $s(7)$ and interpret the results in the context of this problem.

b. What is the slope of the function? Interpret the slope in the context of this problem.

For Exercises 13–23, refer to the graph.



13. Find $f(1)$.

14. Find $f(4)$.

15. Write the domain of f.

16. Write the range of f.

17. True or false? The value $y = 5$ is in the range of f.

18. Find the x-intercept of the function.

19. For what value(s) of x is $f(x) = 0$?

20. For what value(s) of x is $f(x) = 1$?

21. For what open interval(s) is f increasing?

22. For what open intervals is f decreasing?

23. For what open intervals is f constant?

For Exercises 24–27, determine if the function is constant, linear, quadratic, or none of these.

24. $f(x) = -3x^2$

25. $g(x) = -3x$

26. $h(x) = -3$

27. $k(x) = -\dfrac{3}{x}$

28. Find the x- and y-intercepts for $f(x) = \dfrac{3}{4}x + 9$.

29. Write a variation model using k as the constant of variation. The variable x varies directly as y and inversely as the square of t.

30. The period of a pendulum varies directly as the square root of the length of the pendulum. If the period of the pendulum is 2.2 sec when the length is 4 ft, find the period when the length is 9 ft.

Chapters 1–4 Cumulative Review Exercises

1. Solve the equation.

$$\frac{1}{3}t + \frac{1}{5} = \frac{1}{10}(t - 2)$$

2. Simplify. $5 - 3(2 - \sqrt{25}) + 2 - 10 \div 5$

3. Write the inequalities in interval notation.

a. x is greater than or equal to 6.

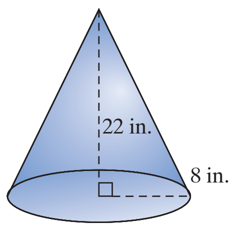
b. x is less than 17.

c. x is between -2 and 3, inclusive.

4. Solve the inequality. Write the solution set in interval notation.

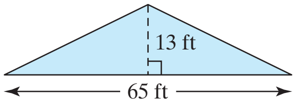
$$4 \le -6y + 5$$

5. Determine the volume of the cone pictured here. Round your answer to the nearest whole unit.



6. Find an equation of the line passing through the origin and perpendicular to $3x - 4y = 1$. Write your final answer in slope-intercept form.

7. Find the pitch (slope) of the roof.



8. **a.** Explain how to find the x- and y-intercepts of a function $y = f(x)$.

b. Find the y-intercept of the function defined by $f(x) = 3x + 2$.

c. Find the x-intercept(s) of the function defined by $f(x) = 3x + 2$.

9. Is the ordered triple (2, 1, 0) a solution to the following system of equations? Why or why not?

$$\begin{aligned} x + 2y - z &= 4 \\ 2x - 3y - z &= 1 \\ -3x + 2y + 2z &= 8 \end{aligned}$$

10. Solve the system by using the substitution method.

$$\begin{aligned} -y - 2x &= -10 \\ 4x - 20 &= -5y \end{aligned}$$

11. Solve the system by using the addition method.

$$\begin{aligned} 5x + 7y &= -9 \\ -3x - 2y &= -10 \end{aligned}$$

12. Solve the system.

$$\begin{aligned} -\frac{1}{4}x + \frac{1}{3}y &= -1 \\ \frac{1}{2}x - \frac{3}{10}y &= 2 \end{aligned}$$

13. One positive number is two-thirds of another positive number. The larger number is 12 more than the smaller. Find the numbers.

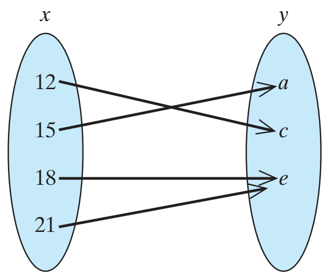
14. State the domain and range of the relation. Is the relation a function? $\{(3, -1), (4, -5), (3, -8)\}$

15. The linear function defined by $N(x) = 420x + 5260$ provides a model for the number of full-time-equivalent (FTE) students attending a community college from 1988 to 2006. Assume that $x = 0$ corresponds to the year 1988.

a. Use this model to find the number of FTE students who attended the college in 1996.

b. If this linear trend continues, predict the year in which the number of FTE students will reach 14,920.

16. State the domain and range of the relation. Is the relation a function?



17. Given: $f(x) = \frac{1}{2}x - 1$ and $g(x) = 3x^2 - 2x$

a. Find $f(4)$. **b.** Find $g(-3)$.

For Exercises 18–19, write the domain of the functions in interval notation.

18. $f(x) = \dfrac{1}{x - 15}$

19. $g(x) = \sqrt{x - 6}$

20. Simple interest varies jointly as the interest rate and as the time the money is invested. If an investment yields \$1120 interest at 8% for 2 years, how much interest will the investment yield at 10% for 5 years?