CCGPS Frameworks Student Edition

Mathematics

CCGPS Coordinate Algebra

Unit 3: Linear and Exponential Functions

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Dr. John D. Barge, State School Superintendent
"Making Education Work for All Georgians"

The contents of this guide were developed under a grant from the U. S. Department of Education. However, those contents do not necessarily represent the policy of the U. S. Department of Education, and you should not assume endorsement by the Federal Government.

Unit 3
Linear and Exponential Functions

Table of Contents

OVERVIEW

In this unit students will:

- Represent and solve linear equations and inequalities graphically using real-world contexts.
- Use function notation.
- Interpret linear and exponential functions that arise in applications in terms of the context.
- Analyze linear and exponential functions and model how different representations may be used based on the situation presented.
- Build a function to model a relationship between two quantities.
- Create new functions from existing functions.
- Construct and compare linear and exponential models and solve problems.
- Interpret expressions for functions in terms of the situation they model.

Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in the unit. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrate the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the **"Strategies for Teaching and Learning"** and the tasks listed under **"Evidence of Learning"** be reviewed early in the planning process.

Webinar Information

A two-hour course overview webinar may be accessed at
http://www.gpb.org/education/common-core/2012/02/28/mathematics-9th-grade

The unit-by-unit webinars may be accessed at
https://www.georgiastandards.org/Common-Core/Pages/Math-PL-Sessions.aspx

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS ADDRESSED

Represent and solve equations and inequalities graphically

MCC9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *(Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.)*

MCC9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, ~~polynomial, rational, absolute value~~, exponential, and ~~logarithmic~~ functions.

Understand the concept of a function and use function notation

MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.)*

Interpret functions that arise in applications in terms of the context

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~. *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers.)*

Analyze functions using different representations

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

MCC9-12.F.IF.7a Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima.

MCC9-12.F.IF.7e Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior, and ~~trigonometric functions, showing period, midline, and amplitude.~~

MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

Build a function that models a relationship between two quantities

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions

MCC9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)*

MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MCC9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function.~~

MCC9-12.F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. *(Limit exponential functions to those of the form $f(x) = b^x + k$.)*

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ENDURING UNDERSTANDINGS

- Linear equations and inequalities can be represented graphically and solved using real-world context.
- Understand the concept of a function and be able to use function notation.
- Understand how to interpret linear and exponential functions that arise in applications in terms of the context.
- When analyzing linear and exponential functions, different representations may be used based on the situation presented.
- A function may be built to model a relationship between two quantities.
- New functions can be created from existing functions.
- Understand how to construct and compare linear and exponential models and solve problems.
- Understand how to interpret expressions for functions in terms of the situation they model.

CONCEPTS AND SKILLS TO MAINTAIN

In order for students to be successful, the following skills and concepts need to be maintained:

- Know how to solve equations, using the distributive property, combining like terms and equations with variables on both sides.
- Know how to solve systems of linear equations.
- Understand and be able to explain what a function is.
- Determine if a table, graph or set of ordered pairs is a function.
- Distinguish between linear and non-linear functions.
- Write linear equations and use them to model real-world situations.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school children. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks**
http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.
http://intermath.coe.uga.edu/dictnary/homepg.asp

Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Arithmetic Sequence.** A sequence of numbers in which the difference between any two consecutive terms is the same.

- **Average Rate of Change.** The change in the value of a quantity by the elapsed time. For a function, this is the change in the *y*-value divided by the change in the *x*-value for two distinct points on the graph.

- **Coefficient.** A number multiplied by a variable in an algebraic expression.

- **Constant Rate of Change.** With respect to the variable x of a linear function $y = f(x)$, the constant rate of change is the slope of its graph.

- **Continuous.** Describes a connected set of numbers, such as an interval.

- **Discrete.** A set with elements that are disconnected.

- **Domain.** The set of *x*-coordinates of the set of points on a graph; the set of *x*-coordinates of a given set of ordered pairs. The value that is the input in a function or relation.

- **End Behaviors.** The appearance of a graph as it is followed farther and farther in either direction.

- **Explicit Expression.** A formula that allows direct computation of any term for a sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$.

- **Exponential Function.** A nonlinear function in which the independent value is an exponent in the function, as in $y = ab^x$.

- **Exponential Model.** An exponential function representing real-world phenomena. The model also represents patterns found in graphs and/or data.
- **Expression.** Any mathematical calculation or formula combining numbers and/or variables using sums, differences, products, quotients including fractions, exponents, roots, logarithms, functions, or other mathematical operations.

- **Even Function.** A function with a graph that is symmetric with respect to the *y*-axis. A function is only even if and only if $f(-x) = f(x)$.

- **Factor.** For any number *x*, the numbers that can be evenly divided into *x* are called factors of *x*. For example, the number 20 has the factors 1, 2, 4, 5, 10, and 20.

- **Geometric Sequence.** A sequence of numbers in which the ratio between any two consecutive terms is the same. In other words, you multiply by the same number each time to get the next term in the sequence. This fixed number is called the common ratio for the sequence.

- **Interval Notation.** A notation representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or brackets are used to show whether the endpoints are excluded or included.

- **Linear Function.** A function with a constant rate of change and a straight line graph.

- **Linear Model.** A linear function representing real-world phenomena. The model also represents patterns found in graphs and/or data.

- **Odd Function.** A function with a graph that is symmetric with respect to the origin. A function is odd if and only if $f(-x) = -f(x)$.

- **Parameter.** The independent variable or variables in a system of equations with more than one dependent variable.

- **Range.** The set of all possible outputs of a function.

- **Recursive Formula.** A formula that requires the computation of all previous terms to find the value of a_n.

- **Slope.** The ratio of the vertical and horizontal changes between two points on a surface or a line.

- **Term.** A value in a sequence--the first value in a sequence is the 1st term, the second value is the 2nd term, and so on; a term is also any of the monomials that make up a polynomial.

- **Vertical Translation.** A shift in which a plane figure moves vertically.

- ***X*-intercept.** The point where a line meets or crosses the *x*-axis

- ***Y*-intercept.** The point where a line meets or crosses the *y*-axis

Scaffolding Task: Talk Is Cheap!

Name_____ Date_____

Mathematical Goals

- Graph linear functions
- Use the graphing calculator to find the intersection of two linear functions
- Interpret the intersection in terms of the problem situation
- Compare functions represented algebraically, graphically, and in tables

Essential Questions

- Why is the concept of a function important
- How do I use function notation to show a variety of situations modeled by functions?
- How do I interpret expressions for functions in terms of the situation they model?

Common Core Georgia Performance Standards

MCC9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *(Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.)*

MCC9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, ~~polynomial, rational, absolute value~~, exponential, and ~~logarithmic~~ functions.

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
5. Use appropriate tools strategically.

Scaffolding Task: Talk Is Cheap!

Name_____ **Date_____**

To encourage communication between parents and their children and to prevent children from having extremely large monthly bills due to additional minute charges, two cell phone companies are offering special service plans for students.

Talk Fast cellular phone service charges $0.10 for each minute the phone is used.
Talk Easy cellular phone service charges a basic monthly fee of $18 plus $0.04 for each minute the phone is used.

Your parents are willing to purchase one of the cellular phone service plans listed above for you. However, to help you become fiscally responsible they ask you to use the following questions to analyze the plans before choosing one.

1. How much would each company charge per month if you talked on the phone for 100 minutes in a month? How much if you talked for 200 minutes in a month?

2. Build a table, make a graph, and write a function rule, $f(x)$ or $g(x)$, to represent the cost of each cellular service in terms of the number of minutes, x.

Table:

Talk Fast:

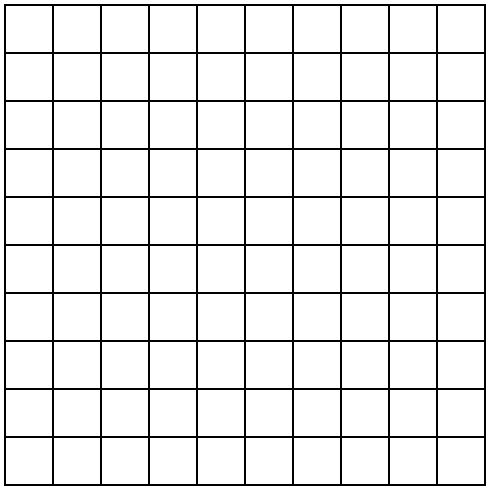
Number of minutes x	0	50	100	150	200	250	300	350
Cost in dollars $f(x)$								

Talk Easy:

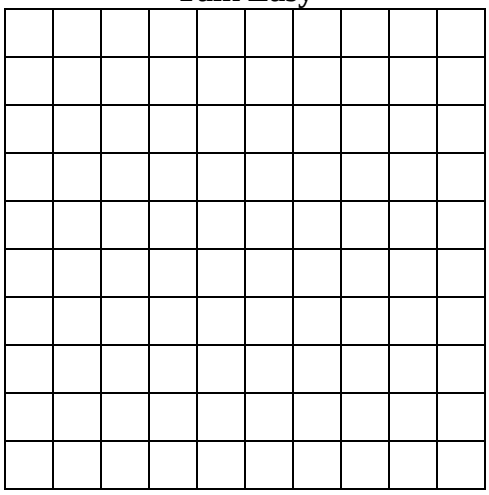
Number of minutes x	0	50	100	150	200	250	300	350
Cost in dollars $f(x)$								

Graph:

Talk Fast



Talk Easy



Function Rule:

Talk Fast: $f(x)$ **Talk Easy:** $g(x)$

Use the table, graph, and/or rule to help answer the following questions:

3. Complete the function notation equation for the given value. Then explain in words what the function notation signifies.

a. $f(50) =$ _____

b. $f(200) =$ _____

c. $f($_____$) = 30$

d. $g(0) =$ _____

e. $g(150) =$ _____

f. $g($_____$) = 20$

4. Which company would be a better financial deal if you plan to use the phone for 200 minutes a month? Explain your reasoning.

5. Which company would be a better financial deal if you plan to use the phone for 500 minutes a month? Explain your reasoning.

6. Depending on the number of minutes you talk on the phone each month, explain to your parents which cellular phone plan is more economical. Include in your explanation the point at which both cellular phone plans cost the same amount of money.

7. If you know the cost of each plan for 300 minutes, can you double this cost to find the cost for 600 minutes? Explain your answer.

Practice Task: Functioning Well

Name_____ **Date_____**

Mathematical Goals

- Understand the domain and range, notation, and graph of a function
- Use function notation
- Interpret statements that use function notation in terms of context
- Recognize that sequences are functions

Essential Questions

- How do I represent real life situations using function notation?

Common Core Georgia Performance Standards

MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

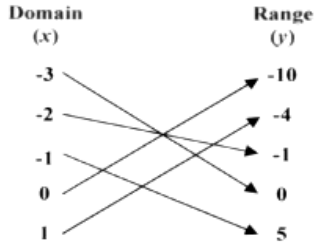
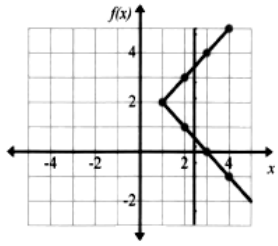
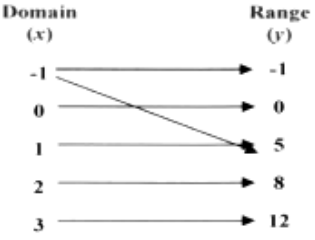
Practice Task: Functioning Well

Name_____ **Date**_____

Consider the definition of a function (A function is a *rule* that assigns each element of set A to a *unique* element of set B. It may be represented as a set of ordered pairs such that no two ordered pairs have the same first member, i.e. each element of a set of inputs (the domain) is associated with a unique element of another set of outputs (the range)).

Part I – Function or Not

Determine whether or not each of the following is a function or not. Write "function" or "not a function" and explain why or why not.

	Relation	**Answer and Explanation**
1.		
2.		
3.		
4.	(x, y) = (student's name, student's shirt color) 	

Part II – Function Notation

Suppose a restaurant has to figure the number of pounds of fresh fish to buy given the number of customers expected for the day. Let $p = f(E)$ where p is the pounds of fish needed and E is the expected number of customers.

5. What would the expressions $f(E + 15)$ and $f(E) + 15$ mean?

6. The restaurant figured out how many pounds of fish needed and bought 2 extra pounds just in case. Use function notation to show the relationship between domain and range in this context.

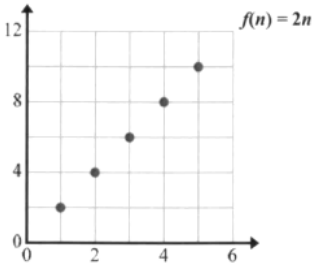
7. On the day before a holiday when the fish markets are closed, the restaurant bought enough fish for two nights. Using function notation, illustrate how the relationship changed.

8. The owner of the restaurant planned to host his 2 fish-loving parents in addition to his expected customers for dinner at the restaurant. Illustrate using function notation

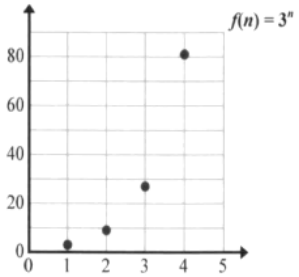
Part III – Graphs are Functions

Write each of the points using function notation.

9.



10.



Skeleton Tower (Short Cycle Task)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=810

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Skeleton Tower*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=279&subpage=expert

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=810

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=811

Mathematical Goals

- Find, extend, and describe mathematical patterns.

Essential Questions

- How do I find, extend, and describe mathematical patterns?

Common Core Georgia Performance Standards

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Standards for Mathematical Practice

- **2.** Reason abstractly and quantitatively.
- **7.** Look for and make use of structure.
- **8.** Look for and express regularity in repeated reasoning.

Background Knowledge

- Students can represent sequences algebraically.

Common Misconceptions

- Students may confuse geometric and arithmetic sequences.
- Students may think about sequences recursively but incorrectly write their pattern as explicit formulas.

Materials

- see FAL website

Grouping

- Individual / small group

Comparing Investments (Formative Assessment Lesson (FAL))

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1250

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website: http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Comparing Investments*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=426&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below: http://map.mathshell.org/materials/download.php?fileid=1250

Mathematical Goals

- Translate between descriptive, algebraic, and tabular data, and graphical representation of a function.
- Recognize how, and why, a quantity changes per unit interval.

Essential Questions

- How do you relate real-life problems to linear or exponential models?

Common Core Georgia Performance Standards

MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MCC9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

MCC9-12.F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. *(Limit exponential functions to those of the form* $f(x) = b^x + k$*.)*

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
4. Model with mathematics.
7. Look for and make use of structure.

Background Knowledge

- Students can compute simple and compound interest.
- Students understand linear and exponential models.

Common Misconceptions

- Students may confuse the formulas and meanings of simple and compound interest.

Essential Questions

- How can I use linear models to decide which of the two payment models is cheaper?

Materials

- see FAL website

Grouping

- Individual / partners

Best Buy Tickets (Short Cycle Task)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=824

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website: http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Best Buy Tickets*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=286&subpage=expert

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=824

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=825

Mathematical Goals

- Students can use linear models to compare two purchasing options.

Essential Questions

- How can I use linear models to decide which of the two payment models is cheaper?

Common Core Georgia Performance Standards

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

Background Knowledge

- Students understand the meaning of slope and *y*-intercept in context when writing linear equations.
- Students may need to know how to solve a system of linear equations, depending on the solution path they follow.

Common Misconceptions

- Students may confuse the slope and the *y*-intercept of a linear equation.
- Students may fail to realize that the answer to the "which is the better buy" question depends on the number of people who attend.

Materials

- see FAL website

Grouping

- Individual / partners

Multiplying Cells (Short Cycle Task)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=788

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website: http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Multiplying Cells*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=268&subpage=apprentice

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=788

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=789

Mathematical Goals

- Use exponential functions to model real-world situations.

Essential Questions

- How can I use exponential functions to model real-world situations?

Common Core Georgia Performance Standards

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MCC9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

MCC9-12.F.LE.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MCC9-12.F.LE.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MCC9-12.F.LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MCC9-12.F.LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function.~~
MCC9-12.F.LE.5	Interpret the parameters in a linear or exponential function in terms of a context. *(Limit exponential functions to those of the form* $f(x) = b^x + k$*.)*

Standards for Mathematical Practice

3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.

Background Knowledge

- Students can work with exponents.
- Students recognize exponential relationships.

Common Misconceptions

- Students may think about sequences recursively but incorrectly write their pattern as explicit formulas.
- Students may interpret 2^3 as $2 \cdot 3$, or they may believe the growth is linear.

Materials

- see FAL website

Grouping

- Individual / partner

Scaffolding Task: You're Toast, Dude!

Name_____ **Date**_____

Mathematical Goals

- Use function notation
- Interpret functions that arise in applications in terms of context
- Analyze functions using different representations
- Build a function that models a relationship between two quantities

Essential Questions

- How do I interpret functions that arise in applications in terms of context?

Common Core Georgia Performance Standards

MCC9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, ~~polynomial, rational, absolute value~~, exponential, and ~~logarithmic~~ functions.

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~. *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.

Scaffolding Task: You're Toast, Dude!

Name_____ **Date**_____

At the You're Toast, Dude! toaster company, the weekly cost to run the factory is $1400 and the cost of producing each toaster is an additional $4 per toaster.

1. Write a function rule representing the weekly cost in dollars, $C(x)$, of producing x toasters.

2. What is the total cost of producing 100 toasters in one week?

3. If you produce 100 toasters in one week, what is the total production cost per toaster?

4. Will the total production cost per toaster always be the same? Justify your answer.

5. Write a function rule representing the total production cost per toaster $P(x)$ for producing x toasters.

6. Using your graphing calculator, create a graph of your function rule from question 5. Use either the graph or algebraic methods to answer the following questions:

a. What is the production cost per toaster if 300 toasters are produced in one week? If 500 toasters are produced in one week?

b. What happens to the total production cost per toaster as the number of toasters produced increases? Explain your answer.

c. How many toasters must be produced to have a total production cost per toaster of $8?

Performance Task: Community Service, Sequences, and Functions

Name_____ Date_____

Mathematical Goals

- Recognize that sequences are functions sometimes defined recursively
- Use technology to graph and analyze functions
- Convert a recursive relationship into an explicit function
- Construct linear and exponential function (including reading these from a table)
- Observe the difference between linear and exponential functions

Essential Questions

- How are sequences and functions related? How can I model one with the other?

Common Core Georgia Performance Standards

MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and gemetric sequences.)*

MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *(Focus on linear and exponential functions.)*

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function.~~

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Standards for Mathematical Practice

4. Model with mathematics.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Performance Task: Community Service, Sequences, and Functions

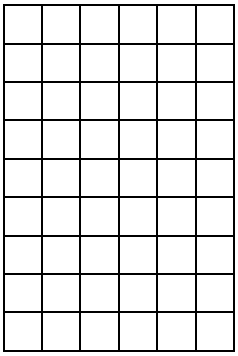
Name_____ **Date**_____

Larry, Moe, and Curly spend their free time doing community service projects. They would like to get more people involved. They began by observing the number of people who show up to the town cleanup activities each day. The data from their observations is recorded in the given table for the Great Four Day Cleanup.

x	***y***
1	5
2	27
3	49
4	71

1. Give a verbal description of what the domain and range presented in the table represents.

2. Sketch the data on the grid below.



3. Determine the type of function modeled in the graph above and describe key features of the graph.

4. Based on the pattern in the data collected, what recursive process could Larry, Curly, and Moe write?

5. Write a linear equation to model the function.

6. How would Larry, Curly, and Moe use the explicit formula to predict the number of people who would help if the cleanup campaign went on for 7 days?

Excited about the growing number of people participating in community service, Larry, Curly, and Moe decide to have a fundraiser to plant flowers and trees in the parks that were cleaned during the Great Four Day cleanup. It will cost them $5,000 to plant the trees and flowers. They decide to sell some of the delicious pies that Moe bakes with his sisters. For every 100 pies sold, it costs Moe and his sisters $20.00 for supplies and ingredients to bake the pies. Larry, Curly, and Moe decide to sell the pies for $5.00 each.

7. Complete the following table to find the total number of pies sold and the amount of money the trio collects.

a. On Day 1, each customer buys the same number of pies as his customer number. In other words the first customer buys 1 pie, the second customer buys 2 pies. Fill in the table showing the number of pies and the amount collected on Day 1. Then calculate the total number of pies sold and dollars collected.

b. Write a recursive and explicit formula for the pies sold on Day 1. Explain your thinking.

Customer Number	***Number of Pies Sold***	***Amount Collected***
1	***1***	***$5***
2	***2***	***$10***
total		

c. On Day 2, the first customer buys 1 pie, the second customer buys 2 pies, the third customer buys 4 pies, the fourth customer buys 8 pies, and so on**.** Complete table based on the pattern established. Then calculate the total number of pies sold and dollars collected.

d. Write a recursive and explicit formula for the pies sold on Day 2. Explain your thinking.

Customer Number	***Number of Pies Sold***	***Amount Collected***
1	***1***	***$5***
2	***2***	***$10***
total		

8. Compare the rates of change on Day 1 and Day 2 for the number of pies sold.

9. Did Larry, Curly, and Moe earn enough in two days to fund their project? Consider costs incurred to bake the pies. Justify your reasoning.

Having Kittens (Formative Assessment Lesson (FAL))

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1204

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website: http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Modeling: Having Kittens*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=407&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1204

Mathematical Goals

- Interpret a situation and represent the constraints and variables mathematically.
- Select appropriate mathematical methods to use.
- Make sensible estimates and assumptions.
- Investigate an exponentially increasing sequence.

Essential Questions

- How can I use mathematical models to determine whether the poster's claim that one cat can have 2000 descendants in just 18 months is reasonable?

Common Core Georgia Performance Standards

MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MCC9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically

Background Knowledge

- The background knowledge required for this task is quite general. There are many entry points to this problem, all of which build from different types of background knowledge.

Common Misconceptions

- Students may forget that each new kitten can also have litters of its own after 4 months.
- Students must make assumptions in order to approach the problem. See discussion in "Solutions" of the FAL.

Materials

- see FAL website

Grouping

- Individual / small group

Learning Task: Building and Combining Functions

Name_____ **Date_____**

Mathematical Goals

- Calculate and interpret rate of change
- Combine functions
- Write explicit function rules

Essential Questions

- How can we use real-world situations to construct and compare linear and exponential models and solve problems?

Common Core Georgia Performance Standards

MCC9-12.F.IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers.)*
MCC9-12.F.LE.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MCC9-12.F.BF.1	Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*
MCC9-12.F.BF.1b	Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
4. Model with mathematics.

Learning Task: Building and Combining Functions

Name_____ **Date**_____

adapted from Functions Modeling Change, A Preparation for Calculus, Connally, Hughes-Hallett, Gleason, et al. John Wiley & Sons, 1998.

Given the functions $r(x) = 4x - 5$ and $s(x) = 3^x$

1. Find $r(x) + s(x)$

2. Find $\frac{r(x)}{s(x)}$.

2. Given the functions f and g as defined in the table below, complete the table.

x	$f(x)$	$g(x)$	$n(x) = f(x) + g(x)$	$p(x) = 2f(x)g(x) - f(x)$	$q(x) = \frac{g(x)}{f(x)}$
1	3	2			
2	4	1			
3	1	4			
4	2	3			

The number of violent crimes committed in major cities is one statistic that is used to determine the safety rating of that city. In this task, we will examine data from two cities to examine the relationships of the crime rates to other factors relative to each city. In Table 1, the number of violent crimes committed in each city is given by year. In Table 2, the population of each city is given by year.

TABLE 1: Number of Violent Crimes

Year	2000	2001	2002	2003	2004	2005
# of Violent Crimes in City A	793	795	807	818	825	831
# of Violent Crimes in City B	448	500	525	566	593	652

4. By just looking at Table 1, which city would you predict is safer? Why?

TABLE 2: Population

Year	2000	2001	2002	2003	2004	2005
Population of City A	61,000	62,100	63,220	64,350	65,510	66,690
Population of City B	28,000	28,588	29,188	29,801	30,427	31,066

5. By just looking at Table 2, which city would you predict is safer? Why?

6. How might these two data sets be related?

Let's define functions to represent the data we have. Let $C(t)$ be the function that represents the number of crimes t years after 2000.

That means that for city A, $C(0)$ = _____, $C(1)$ = _____, and $C(4)$ = _____.

Let $P(t)$ be the function that represents the population in t year, where t is measured in number of years since 2000.

That means that for city A, $P(0)$ = _____, $P(1)$ = _____, and $P(4)$ = _____.

7. We have just identified another notational issue. How can we adjust our notation to indicate the city to which we are referring?

8. Since the independent variable in our data is time, notice that each function written is dependent upon time. That means for us to find the per capita (per person) crime rate for each city, we need the ratio of these two functions. Let $R_A(t)$ be the per capita crime rate in city A and $R_B(t)$ be the per capita crime rate in city B. Using $C(t)$ and $P(t)$ for the appropriate cities, write the functional rule for $R_A(t)$ and $R_B(t)$.

9. Now that you have the two functions R_A and R_B defined, complete the table below showing the per capita violent crime rate in both cities by year using the data from Table 1 and 2. Write each of the function values as a percent.

Year	2000	2001	2002	2003	2004	2005
***t* (years since 2000)**						
$R_A(t)$						
$R_B(t)$						

10. Now, using this data, which city is safer? Why?

11. Make any conclusions about the trends you see in the data. Specifically address rates of change.

12. Write a function rule for $C_A(t)$, $C_B(t)$, $P_A(t)$, $P_B(t)$, and then using these function rules, write an explicit function rule for $R_A(t)$ and $R_B(t)$. Verify that each function gives the correct value that you calculated from the data in the table above.

13. Using the functions, can you make predictions about crime rates in the future if the trends in the given data continue?

Interpreting Functions (Short Cycle Task)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=840

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Interpreting Functions*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=294&subpage=novice

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=840

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=841

Mathematical Goals

- Interpret graphs of functions in context.

Essential Questions

- How can I relate graphs to the context they represent?

Common Core Georgia Performance Standards

MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.)*

MCC9-12.F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~. *(Focus on linear and exponential functions.)*
MCC9-12.F.IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *(Focus on linear and exponential functions.)*
MCC9-12.F.IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers.)*
MCC9-12.F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*
MCC9-12.F.IF.7a	Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima.
MCC9-12.F.IF.7e	Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior, and ~~trigonometric functions, showing period, midline, and amplitude.~~
MCC9-12.F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
6. Attend to precision.

Background Knowledge

- Students have experience relating graphs to real-life situations.

Common Misconceptions

- Students may believe that the vertical line, rather than the horizontal line, shows that the car is not moving.

Materials

- see FAL website

Grouping

- Individual / partner

Practice Task: High Functioning

Name_____ **Date_____**

Mathematical Goals

- Use graphs of vertical translations to determine function rules.
- Relate vertical translations of a linear function to its *y*-intercept.
- Identify even and odd functions

Essential Questions

- How are functions affected by adding or subtracting a constant to the function?
- How does the vertical translation of a linear function model translations for other functions?

Common Core Georgia Performance Standards

MCC9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)*

Standards for Mathematical Practice

3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and express regularity in repeated reasoning.

Practice Task: High Functioning

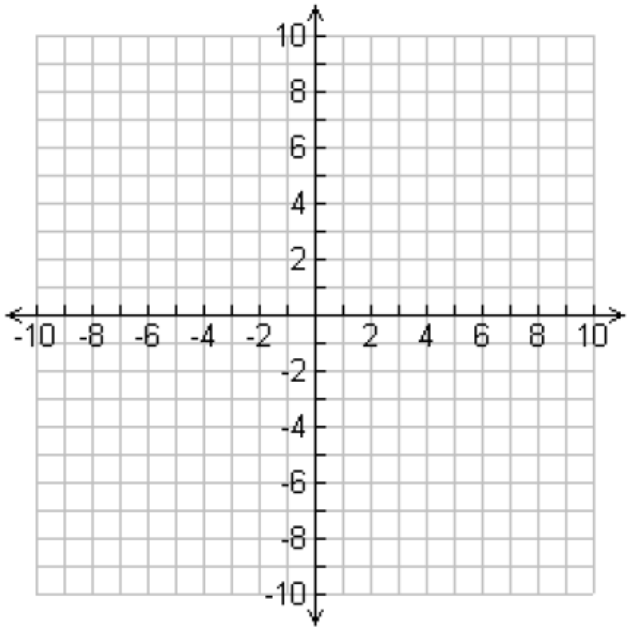
Name_____ **Date**_____

1. Graph and label the following functions.

$f(x) = 2x - 1$

$g(x) = 2x - 7$

$h(x) = 2x + 8$



2. What observations can you make about the three functions? Be sure to include observations about the characteristics and the location of the functions.

3. Analyze specifically what happens to the y-intercepts $f(0)$, $g(0)$, and $h(0)$ in the three functions. How does the *y*-intercept change from…

a. $f \rightarrow g$?

b. $g \rightarrow h$?

c. $f \rightarrow h$?

d. $h \rightarrow f$?

4. Find…

a. $f(1)$

b. $g(1)$

c. $h(1)$

5. What changes in the output as you go from…

a. $f(1) \rightarrow g(1)$?

b. $g(1) \rightarrow h(1)$?

c. $f(1) \rightarrow h(1)$?

d. $h(1) \rightarrow f(1)$?

6. Comparing your answers to 3 and 5, what predictions can you make about other inputs?

7. Write an algebraic rule for the following shifts.

a. $f(x) \rightarrow g(x)$

b. $g(x) \rightarrow h(x)$

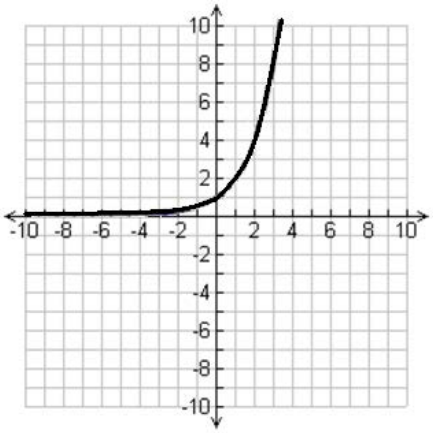
c. $f(x) \rightarrow h(x)$

d. $h(x) \rightarrow f(x)$

8. Write a general rule for a vertical translation.

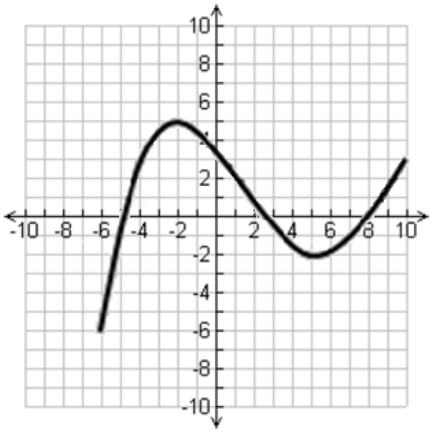
Using the functions below, draw and label the given translations.

9.



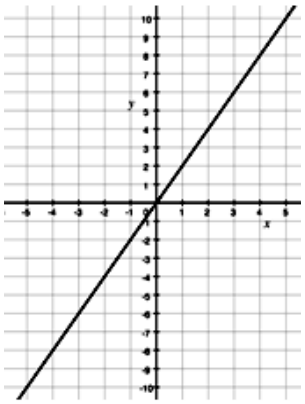
a. $g(x) = f(x) - 4$
b. $h(x) = f(x) + 2$
c. $j(x) = f(x) + 7$

10.



a. $g(x) = f(x) + 3$
b. $h(x) = f(x) - 5$
c. $j(x) = f(x) - 2$

11. The graph of the **odd function** $f(x) = 2x$ is shown below. Fill in the table.

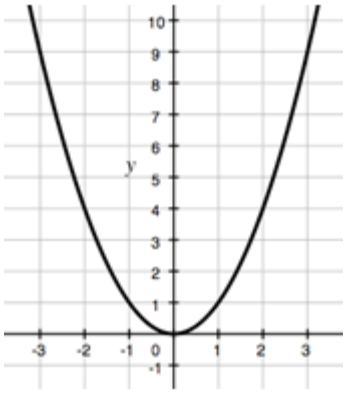


$f(-1) =$	$f(1) =$
$f(-4) =$	$f(4) =$
$f(-8) =$	$f(8) =$
$f(-25) =$	$f(25) =$

12. What characteristics do you notice about odd functions based on the points in the table?

13. Now rotate your paper 180˚ so that the graph is upside down. What further observation can you make about characteristics of odd functions?

14. The graph of the **even function** $g(x) = x^2$ is shown below. Fill in the table.



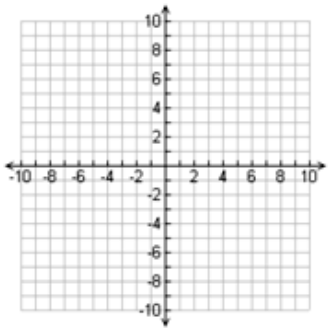
$g(-1) =$	$g(1) =$
$g(-2) =$	$g(2) =$
$g(-3) =$	$g(3) =$
$g(-4) =$	$g(4) =$

15. What characteristics do you notice about even functions based on the table?

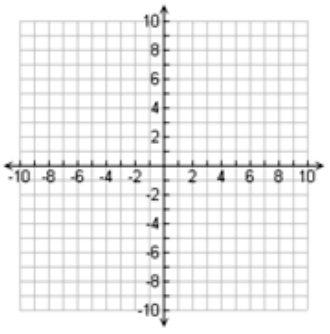
16. Fold the graph of $g(x)$ along the y-axis. What further observations can you make about the characteristics of even functions?

17. Graph the three functions below and explain why they are neither even nor odd.

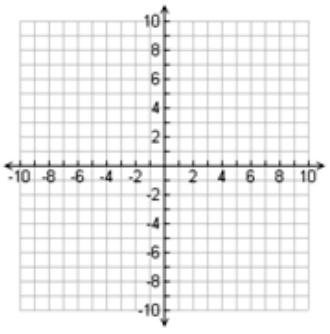
$f(x) = 2x - 6$



$g(x) = 4x + 1$



$h(x) = 2^x$



18. Now demonstrate algebraically that the three functions are neither even nor odd by using the inputs 2 and -2.