CCGPS Frameworks Student Edition

Mathematics

8th Grade
Unit 5: Linear Functions

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"Making Education Work for All Georgians"

Unit 5
LINEAR FUNCTIONS

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OVERVIEW

In this unit students will:

- graph proportional relationships;
- interpret unit rate as the slope;
- compare two different proportional relationships represented in different ways;
- use similar triangles to explain why the slope is the same between any two points on a non-vertical line;
- derive the equation $y = mx$ for a line through the origin;
- derive the equation $y = mx + b$ for a line intercepting the vertical axis at b; and
- interpret equations in $y = mx + b$ form as linear functions.

This unit focuses on extending the understanding of ratios and proportions. Unit rates have been explored in Grade 6 as the comparison of two different quantities with the second unit a unit of one, (unit rate). In seventh grade unit rates were expanded to complex fractions and percents through solving multi-step problems such as: discounts, interest, taxes, tips, and percent of increase or decrease. Proportional relationships were applied in scale drawings, and students should have developed an informal understanding that the steepness of the graph is the slope or unit rate. Now unit rates are addressed formally in graphical representations, algebraic equations, and geometry through similar triangles.

Distance time problems are notorious in mathematics. In this unit, they serve the purpose of illustrating how the rates of two objects can be represented, analyzed and described in different ways: graphically and algebraically. Students create representative graphs and the meaning of various points. They then compare the same information when represented in an equation.

By using coordinate grids and various sets of three similar triangles, students prove that the slopes of the corresponding sides are equal, thus making the unit rate of change equal. After proving with multiple sets of triangles, students generalize the slope to $y = mx$ for a line through the origin and $y = mx + b$ for a line through the vertical axis at b.

In Grade 8, the focus, of course, is on linear functions, and students begin to recognize a linear function from its form $y = mx + b$. Students also need experiences with nonlinear functions, including functions given by graphs, tables, or verbal descriptions but for which there is no formula for the rule, such as a girl's height as a function of her age. Students learn that proportional relationships are part of a broader group of linear functions, and they are able to identify whether a relationship is linear. Nonlinear functions are included for comparison. Later, in high school, students use function notation and are able to identify types of nonlinear functions.

In the elementary grades, students explore number and shape patterns (sequences), and they use rules for finding the next term in the sequence. At this point, students describe

sequences both by rules relating one term to the next and also by rules for finding the *n*th term directly. (In high school, students will call these recursive and explicit formulas.) Students express rules in both words and in symbols. Instruction focuses on additive and multiplicative sequences as well as sequences of square and cubic numbers, considered as areas and volumes of cubes, respectively. Students compute the area and perimeter of different-size squares and identify that one relationship is linear while the other is not by either looking at a table of value or a graph in which the side length is the independent variable (input) and the area or perimeter is the dependent variable (output).

When plotting points and drawing graphs, students develop the habit of determining, based upon the context, whether it is reasonable to “connect the dots” on the graph. In some contexts, the inputs are discrete, and connecting the dots can be misleading.

Students examine the graphs of linear functions and use graphing calculators or computer software to analyze or compare at least two functions at the same time. Illustrate with a slope triangle where the run is "1" that slope is the "unit rate of change." Compare this in order to compare two different situations and identify which is increasing/decreasing at a faster rate.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under “Evidence of Learning” be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

Understand the connections between proportional relationships, lines, and linear equations.

MCC8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

MCC8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Define, evaluate, and compare functions.

MCC8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

RELATED STANDARDS

Use properties of operations to generate equivalent expressions.

MCC7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

MCC7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

MCC7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations as strategies to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

MCC7.EE.4 Use variables to represent quantities in a real world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

MCC7.EE.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
In grade 8, students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"

2 Reason abstractly and quantitatively.
In grade 8, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

3 Construct viable arguments and critique the reasoning of others.
In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.

4 Model with mathematics.
In grade 8, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and

compare properties of functions provided in different forms. Students use scatter plots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

5 Use appropriate tools strategically.
Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.

6 Attend to precision.
In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.

7 Look for and make use of structure.
Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.

8 Look for and express regularity in repeated reasoning.
In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

ENDURING UNDERSTANDINGS

- Patterns and relationships can be represented graphically, numerically, and symbolically.
- Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- determining unit rate
- applying proportional relationships
- recognizing a function in various forms
- plotting points on a coordinate plane
- understanding of writing rules for sequences and number patterns
- differences in graphing of discrete and continuous data
- attributes of similar figures

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

Visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see additional definitions and specific examples of many terms and symbols used in grade 8 mathematics.

- **Intersecting Lines:** Two lines that cross each other. Lines intersect at one point unless the lines fall directly on top of each other (in which case they are essentially the same line and are sometimes called coincidental).

- **Origin:** The point of intersection of the vertical and horizontal axes of a Cartesian plane. The coordinates of the origin are (0, 0).

- **Proportional Relationships:** A relationship between two equal ratios.

- **Slope:** The "steepness" of a line. The slope of a line can be found directly when a linear equation is in slope-intercept form ($y = mx + b$). In this form, the slope is the coefficient of x and is represented by the letter m. The slope of a line can also be found by determining the ratio of the "rise" to the "run" between two points on the graph. In other words, slope measures how much the line rises vertically given a particular run or horizontal distance.

- **Unit Rate**: A comparison of two measurements in which the second term has a value of 1. Unit rates are used to compare the costs of items in a grocery store.

TASK: By the Book

The thickness of book manuscripts depends on the number of pages in the manuscript. On your first day working at the publishing house, you are asked to create tools editors can use to estimate the thickness of proposed manuscripts. After experimenting with the different weights of paper, you discover that 100 sheets of paper averages 1.25 cm thick.

Your task is to create a table of possible pages and the corresponding thickness. This information should also be presented in a graph. Since you cannot include all the possible number of pages, you also need a formula that editors can use to determine the thickness of any manuscript.

1. Based on the request, determine the variables in the relationship, then identify the independent variable and the dependent variable. Add them to the table below.

   Independent variable -

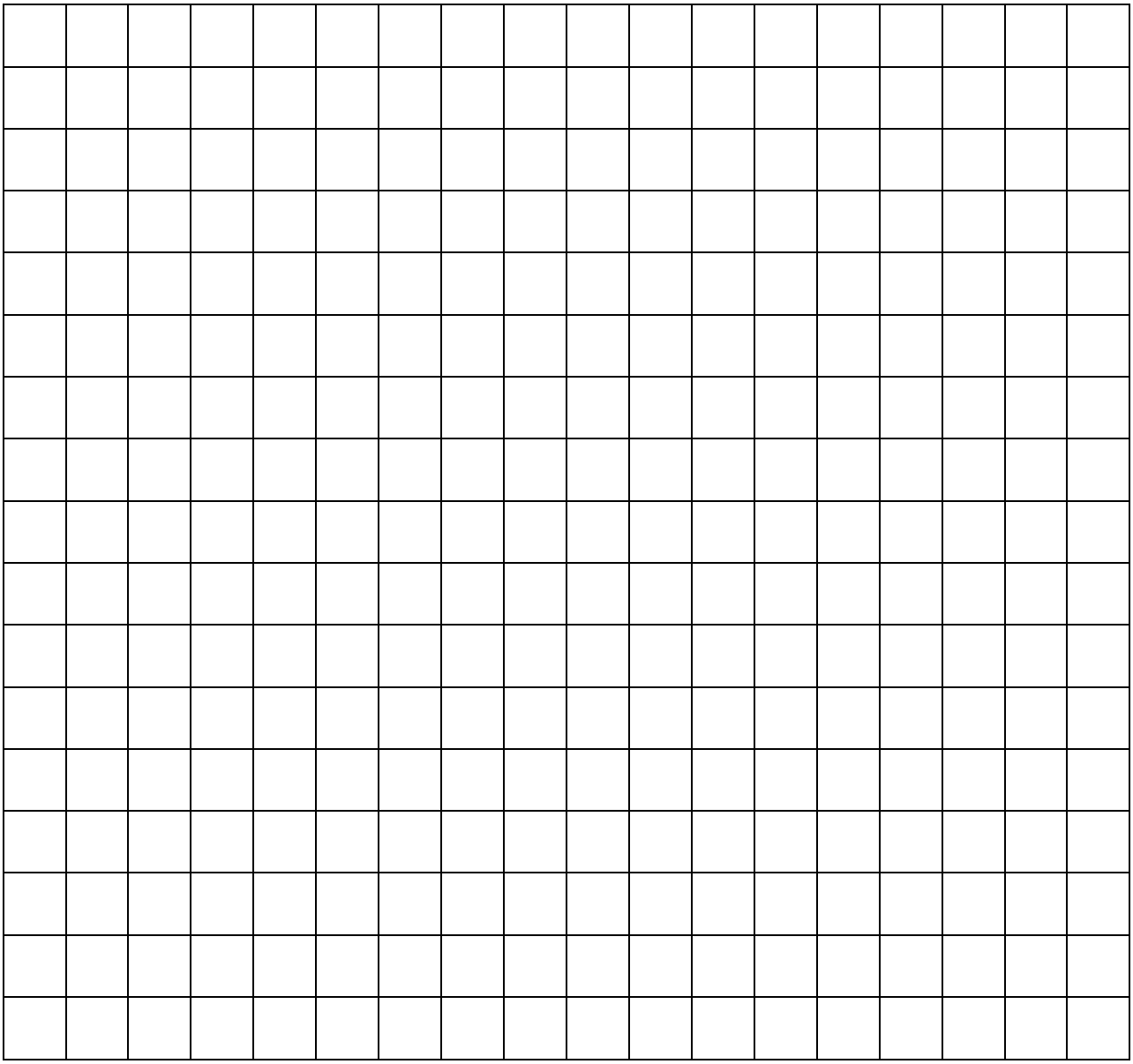
   Explain your reasoning.

   Dependent variable -

   Explain your reasoning.

2. Complete the table for manuscripts from zero to 500 in 50 page intervals.

3. Use the information from the table in problem 2 to create a graph.



4. Write an equation for the thickness of a manuscript given any number of pages it might contain.

The copyrighters for the publisher's website also asked that the information be represented for them so they can tell how many pages are in a book based on its thickness.

5. Based on the request, determine the variables in the relationship, the independent variable, and the dependent variable.

   Independent variable -

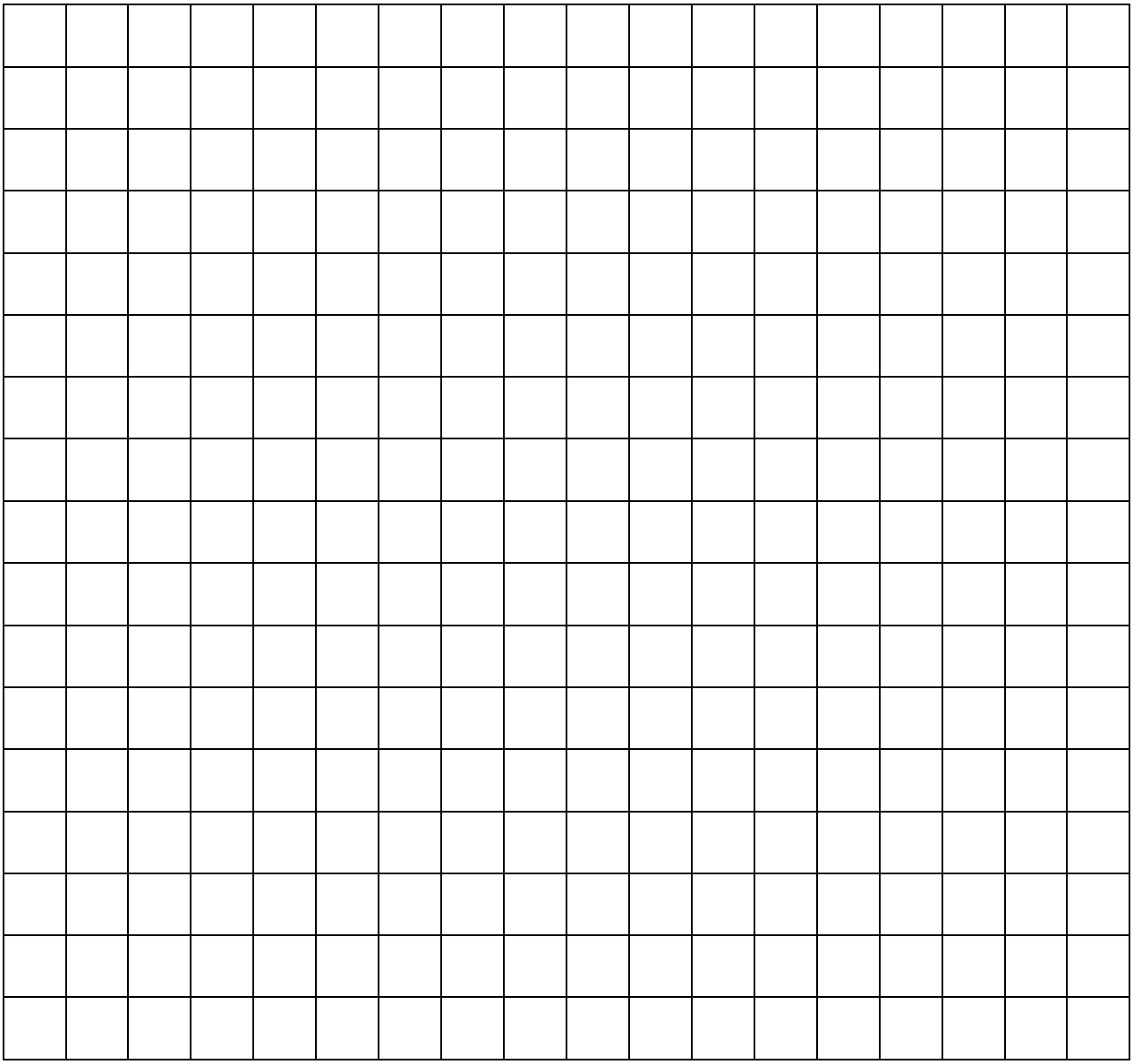
   Explain your reasoning.

   Dependent variable -

   Explain your reasoning.

6. Using the information from problem 5, Create a table for manuscripts from zero to 10 in 2 cm intervals.

7. Use the information from your table in problem 6 to create a graph.



8. Write an equation for the number of pages a manuscript will contain given its thickness.

Based on your findings, answer the following questions.

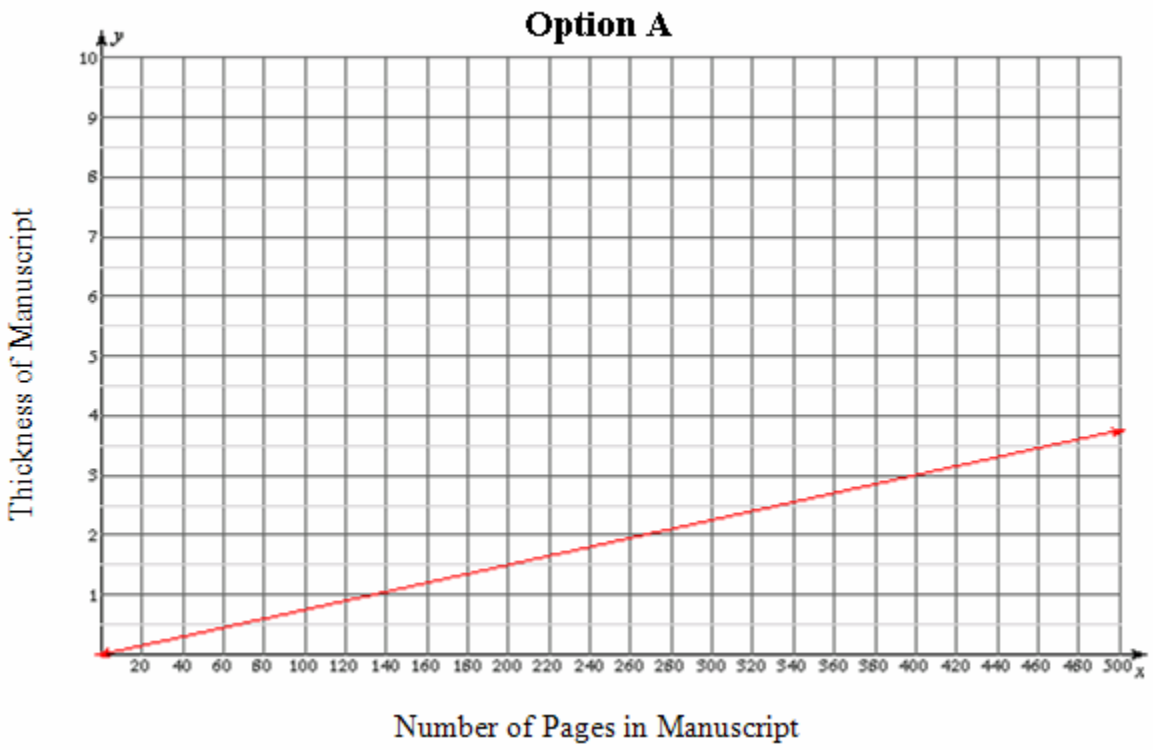
9. What is the unit rate for sheets of paper per centimeter? Explain how you know.

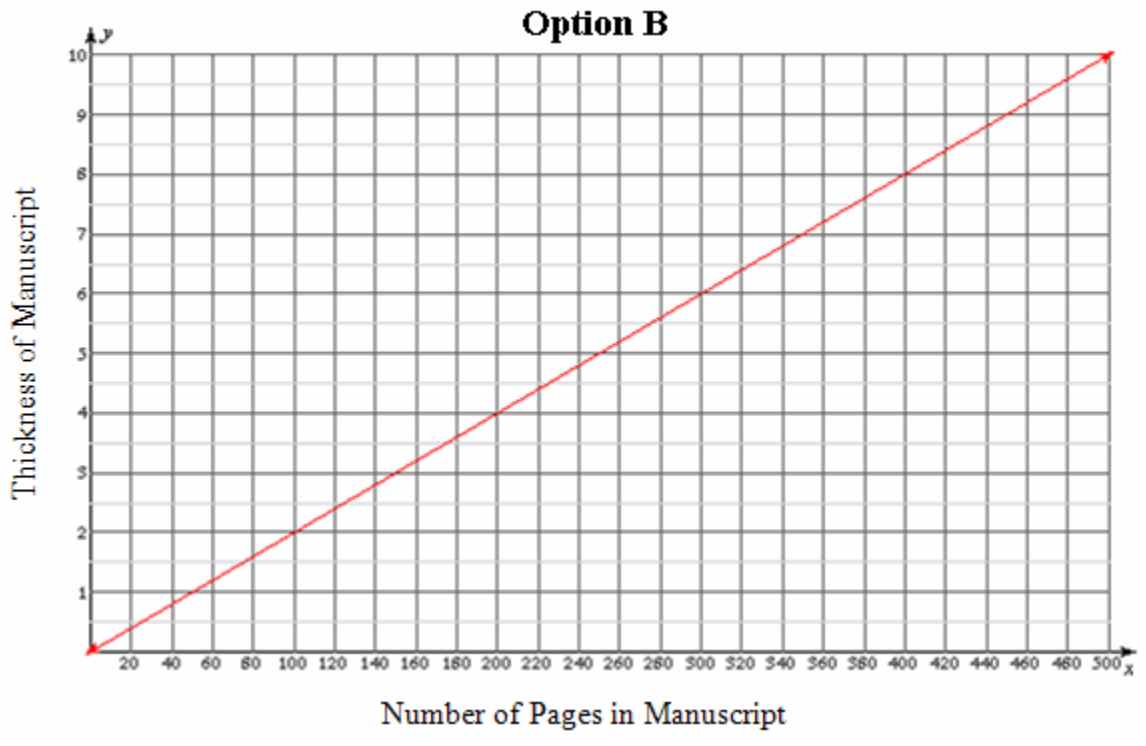
10. What is the unit rate for height for sheet of paper? Explain how you know.

11. How many sheets of paper will be in a manuscript 4 cm thick? Explain how you know.

12. How thick will a 300 page manuscript be? Explain how you know.

The publishing company offers two other types of paper for printing. The graphs below show the thickness of manuscripts based on the number of pages for two different options of paper.





13. Use the graphs to determine the slope of the line for each option.

    Option A:

    Option B:

14. Express each of these slopes as a unit rate for the thickness of the manuscripts.

    Option A:

    Option B:

15. Write an equation for each paper option.

    Option A:

    Option B:

16. Explain how the unit rate/slope is evident in each equation and graph.

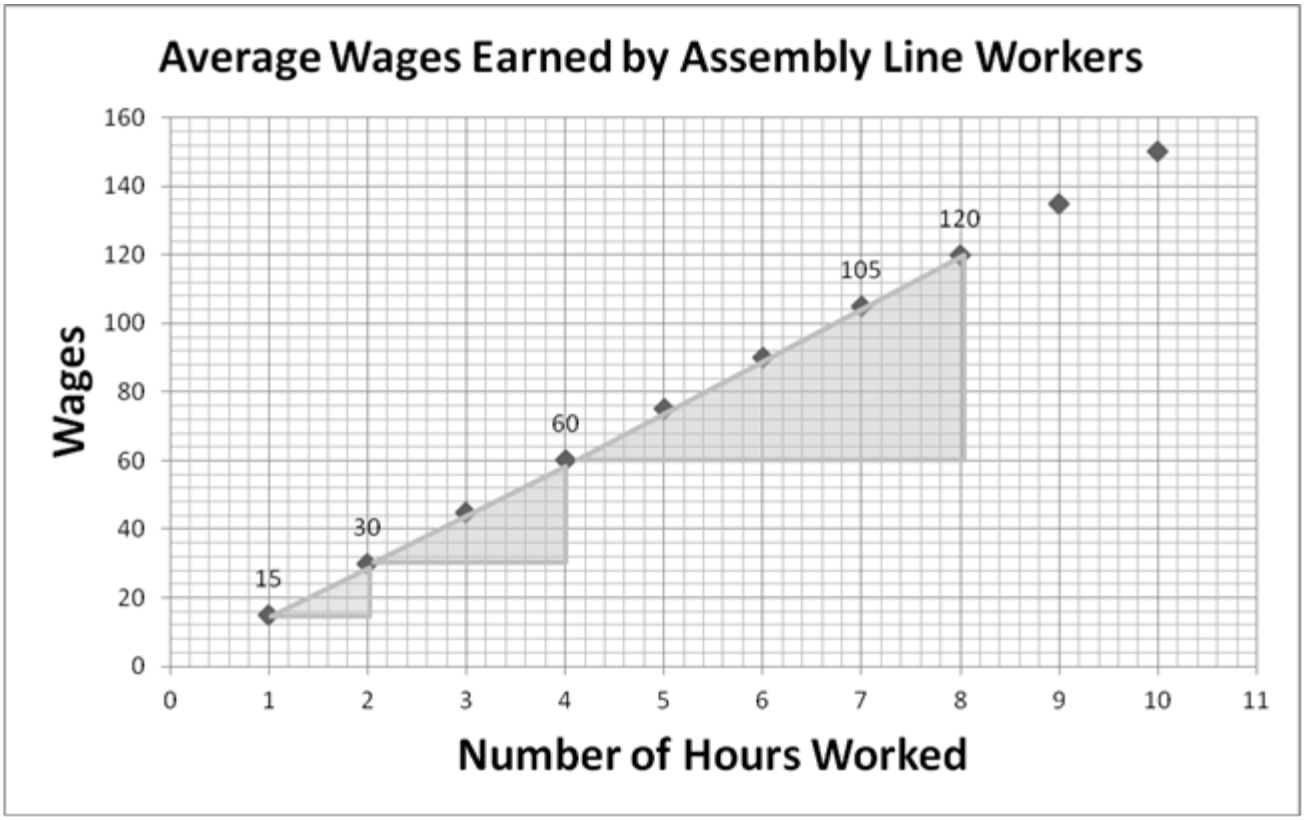
    Option A:

    Option B:

TASK: What's My Line?

Part 1: Average Wages

The data shown in the graph below reflects average wages earned by assembly line workers across the nation.



a. What hourly rate is indicated by the graph? Explain how you determined your answer.

b. What is the ratio of the height to the base of the small, medium and large triangles? What patterns do you observe? What might account for those patterns?

c. The slope of a line is found by forming the ratio of the change in y to the change in x between any two points on the line. What is the slope of the line formed by the data points in the graph above? Explain how you know.

d. Write an equation for the earnings of the average assembly line worker.

e. According to the graph and equation, in a 40-hour week, how much will the average assembly line worker earn? How do you know?

f. With changes in the economy, the average wages can change. How would a decrease of $2 in the average change the equation and graph?

g. How would an increase $5 in the average change the equation and graph?

Part 2: Comparing Wages

The average hourly wages of different jobs are presented below. Using the information provided for each of the different jobs, compare the average hourly wage of these jobs with that of the assembly line worker in Part 1. For each job, include the hourly wage (unit rate), wages earned for 40 hours, and number of hours worked needed to earn $100.

Plumber

$W = \$20h$, where W represents the wages earned and h represents the hours worked.

Machinist

Hours worked	Wages Earned
1	$ 18.50
2	$ 37.00
3	$ 55.50
4	$ 74.00
5	$ 92.50
6	$ 111.00
7	$ 129.50
8	$ 148.00
9	$ 166.50
10	$ 185.00