Linear and Exponential Functions **Name: _________________**

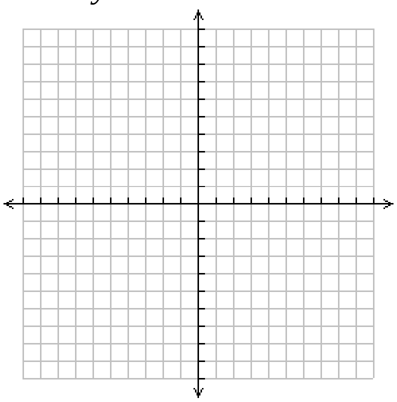
Date: ________________

Represent and solve equations and inequalities graphically
MCC912.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *(Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.)*
MCC9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Understand the concept of a function and use function notation
MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. *(Draw examples from linear and exponential functions.)*
MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*
MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.)*
Interpret functions that arise in applications in terms of the context
MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. *(Focus on linear and exponential functions.)*
MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *(Focus on linear and exponential functions.)*
MCC9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers.)*
Analyze functions using different representations
MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*
MCC9-12.F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.
MCC9-12.F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*
Build a function that models a relationship between two quantities
MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. ★*(Limit to linear and exponential functions.)*
MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*
MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*
MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
Build new functions from existing functions
MCC9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)*
Construct and compare linear, quadratic, and exponential models and solve problems
MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.
MCC9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
Interpret expressions for functions in terms of the situation they model
MCC9-12.F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. ★*(Limit exponential functions to those of the form $f(x) = bx + k$.)*

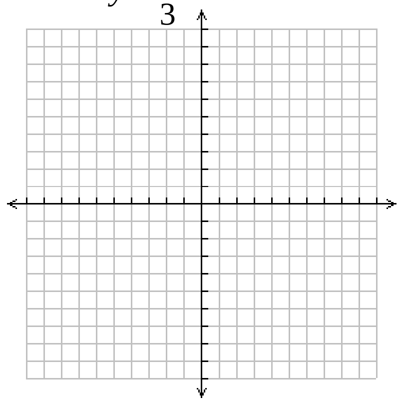
LESSON 3.0 ALGEBRA EXERCISES

Plot the equation

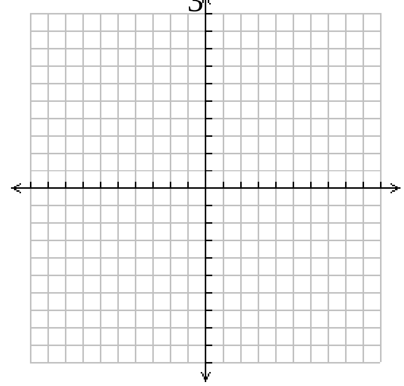
1. $y = 2x + 3$



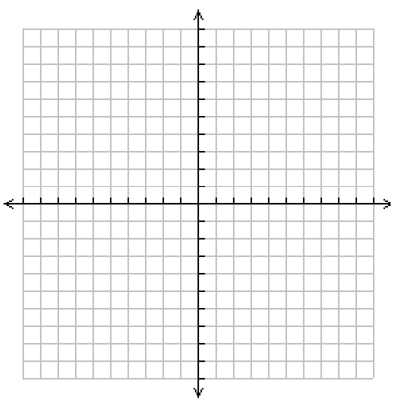
2. $y = \frac{1}{3}x - 6$



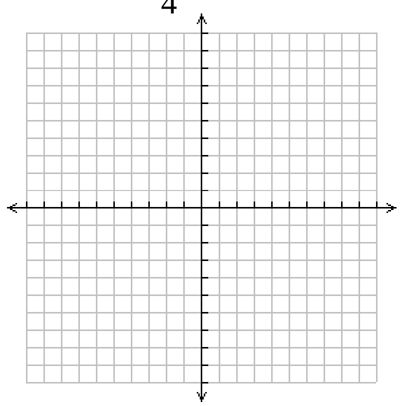
3. $y = -\frac{2}{3}x + 3$



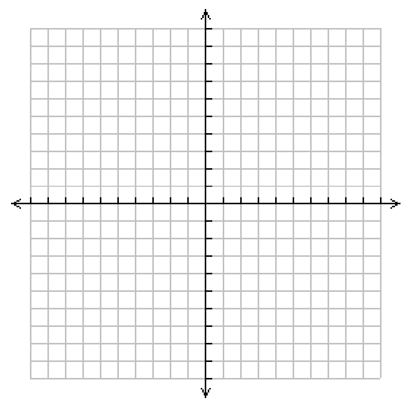
4. $2x + 3y = 6$



5. $y = \frac{x}{4} - 5$

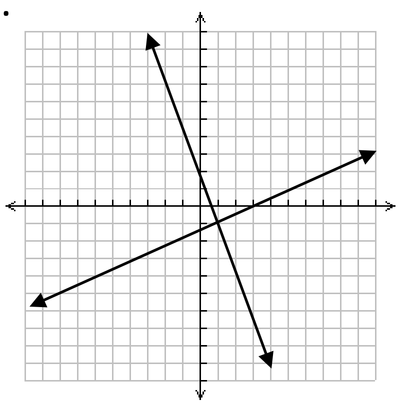


6. $y = -3$



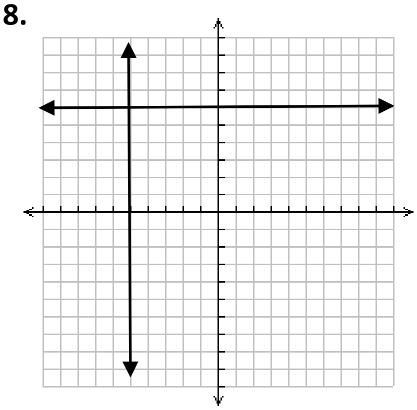
What is the solution for x and y:

7.



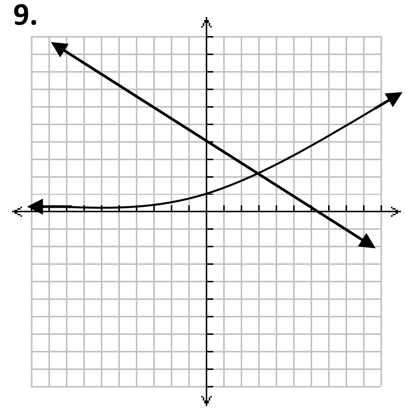
X=_______ Y=________

8.



X=_______ Y=________

9.



X=_______ Y=________

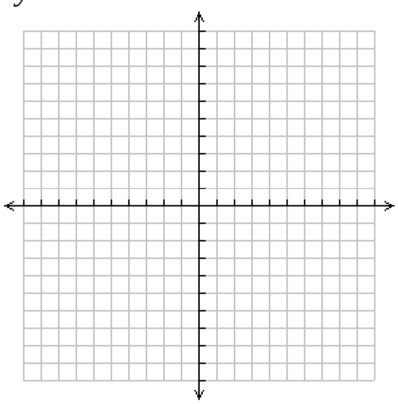
Solve for x and y

9. $y = 3x + 1$
$y = -x + 5$

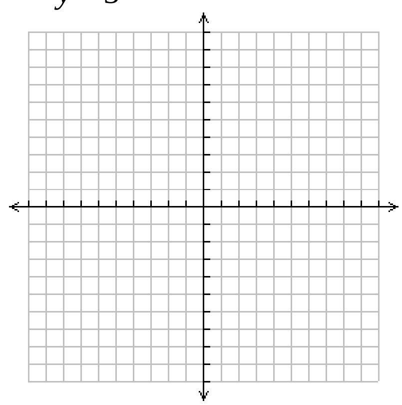
10. $y = x$
$y = 5$

Plot both graphs and determine the solution for x and y. Use a graphing calculator.

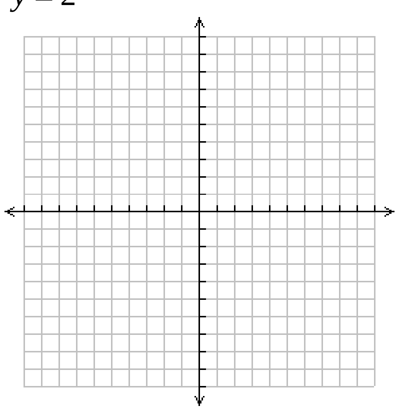
11. $y = 3x + 1$
$y = 2^x$



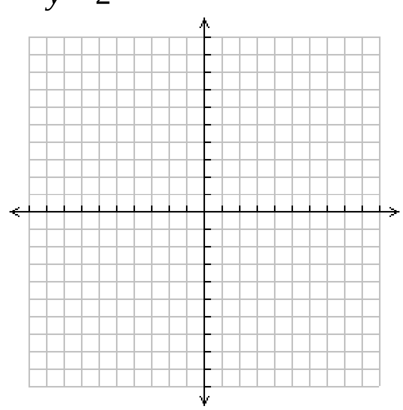
12. $y = -x$
$y = 3^x$



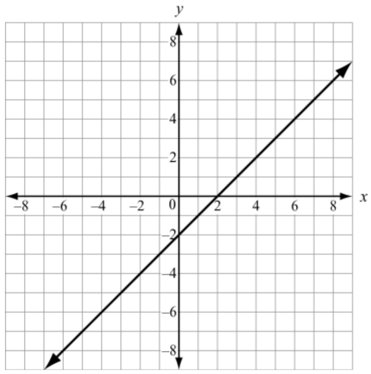
13. $y = x - 2$
$y = 2^{x+1}$



14. $y = x$
$y = 2^{-x}$



15. Which equation corresponds to the graph shown?



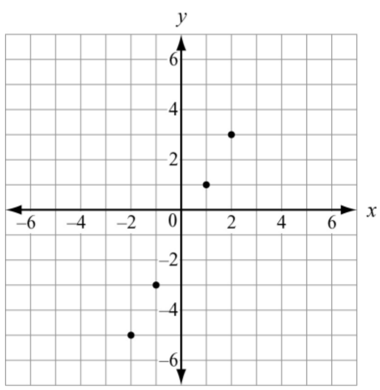
A. $y = x + 1$

B. $y = 2x + 1$

C. $y = x - 2$

D. $y = 3x - 1$

16. Which equation corresponds to the points in the coordinate plane?



A. $y = 2x - 1$

B. $y = x - 3$

C. $y = x + 1$

D. $y = x - 1$

17. Based on the tables, what common point do the equations $y = {-x} + 5$ and $y = 2x - 1$ share?

y=-x+5	
x	y
-1	6
0	5
1	4
2	3
3	2

y=2x-1	
x	y
-1	-3
0	-1
1	1
2	3
3	5

A. (1, 1)

B. (3, 5)

C. (2, 3)

D. (3, 2)

Lesson 3.1 Represent Functions as Rules and Tables

Vocabulary

A **function** consists of:

A **set** called the **domain** containing numbers called **inputs** and a **set** called the **range** containing numbers called **outputs**.

The **input** is called the **independent variable**.

The **output** is called the **dependent variable**. The **output** (dependent variable) is dependent on the value of the **input variable**.

Example: **Identify the domain and range of a function**

The input-output table represents the price of various lobsters at a fish market. Identify the domain and range of the function.

Input (pounds)	1.4	2.2	3.2	4.3	5.1	5.3
Output (dollars)	$7.60	$10.90	$16.10	$20.50	$25.70	$26.90

Solution: The domain is the set of inputs: 1.4, 2.2, 3.2, 4.3, 5.1, 5.3.

The range is the set of outputs: 7.60, 10.90, 16.10, 20.50, 25.70, 26.90.

PROBLEMS

Identify the domain and range of the function.

1.

Input	1	3	5	7
Output	0	2	4	6

2.

Input	0	2	6	9
Output	0	1	3	5

3.

Input	5	7	15	17
Output	3	2	1	8

Example 2 **Make a table for a function**

The domain of the function y = x + 2 is 0, 2, 5, 6. Make a table for the function then identify the range of the function.

x	0	2	5	6
y=x+2	**0+2=2**	**2+2=4**	**5+2=7**	**6+2=8**

The range of the function is 2, 4, 7, 8.

PROBLEMS

Make a table for the function. Identify the range of the function.

1. y = 2x -1 **Domain = 0, 1, 3, 5**

x				
y=2x-1				

The range of the function is:

2. y = -4x + 3 **Domain = -2, 2, 4, 5**

x				
y=-4x+3				

The range of the function is:

3. y = 0.5x – 3 **Domain = 0, 1, 2, 3**

x				
y=0.5x-3				

The range of the function is:

4. y = ½ x – 2 **Domain = 2, 4, 8, 10**

x				
y=½ x – 2				

The range of the function is:

Example 3 **Write a rule for the function**

Input	2	4	6	8
Output	6	12	18	24

Solution Let x be the input and y be the output. Realize that the output is 3 times the input. Therefore, a rule for the function is $y = 3x$.

PROBLEMS

Write a rule for the function

1.

Input	3	6	7	8
Output	15	30	35	40

2.

Input	2	4	6	8
Output	3	5	7	9

3.

Input	1	2	3	4
Output	6	5	4	3

4.

Input	1	2	3	4
Output	4	5	6	7

5. These tables show points from two linear functions.

Function 1

x	$f(x)$
1	5
2	7
3	9
4	11

Function 2

x	$f(x)$
1	–1
2	2
3	5
4	8

Which of these linear functions has a slope **greater** than the slope for function 1 and **less** than the slope for function 2?

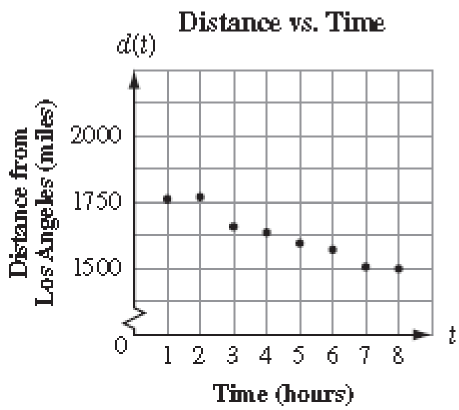
A. $f(x) = 1.5x + 1$

B. $f(x) = 2x + 2.5$

C. $f(x) = 2.5x - 6$

D. $f(x) = 3x + 2$

6. A train traveled from Chicago to Los Angeles. The points graphed on this coordinate grid show the distance the train was from Los Angeles after each of the first eight hours of the trip.



The linear regression model for the data is d(t)=−58t+1888. Based on the regression function, which is the best estimate for the total time it took the train to travel the entire distance? (Hint: the distance travelled is equal to zero once the train has travelled the total distance. You basically trying to find the x-intercept of the graph)

A. 10 hours

B. 33 hours

C. 53 hours

D. 63 hours

Lesson 3.2 Represent Functions as Graphs

Example 1 **Graph a function**

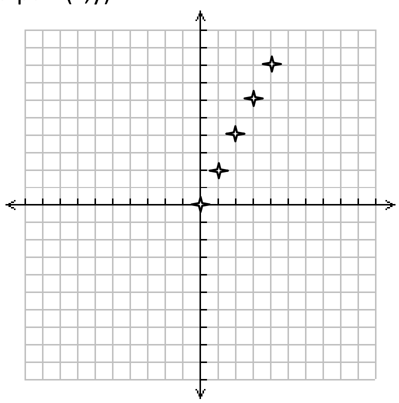
Graph the function y=2x with domain 0, 1, 2, 3, and 4.

Solution

Step 1: Make an input-output table.

X	0	1	2	3	4
y	0	2	4	6	8

Step 2: Plot a point for each ordered pair (x,y).

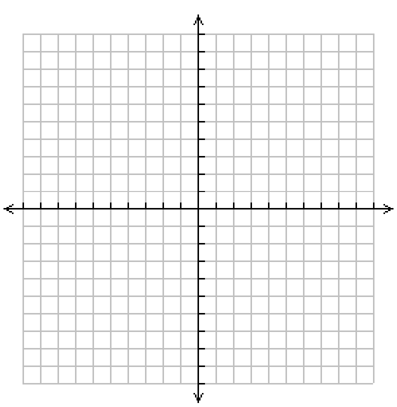


PROBLEMS

Graph the function

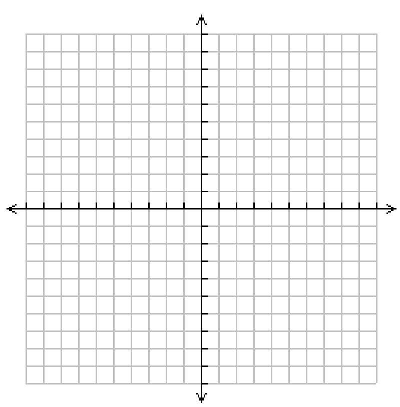
1. $y = 3x$ **Domain: 0, 1, 2, 3**

X	0	1	2	3
y				



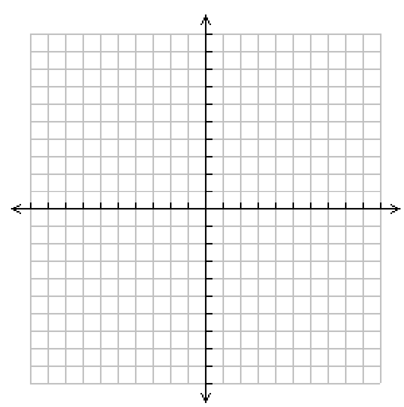
2. $y = \frac{1}{2}x$ **Domain: 0, 2, 4, 6**

X	0	2	4	6
y				



3. $y = 2^x - 2$ **Domain: 0, 1, 2, 3**

X	0	1	2	3
y				



Lesson 3.3 Graph Using Intercepts

Vocabulary	The x-coordinate of a point where the graph crosses the x-axis is the x-intercept. The y-coordinate of a point where the graph crosses the y-axis is the y-intercept.

Example 1 **Find the intercepts of a graph of an equation.**

Find the x-intercept and the y-intercept of the graph 4x – 8y = 24

Solution: To find the x intercept, replace y with a 0 and solve for x

4x – 8(0) = 24

Solve for x = 6

To find the y intercept, replace x with a 0 and solve for y

4(0) – 8y = 24

Solve for y = -3

Therefore, the x-intercept is 6. The y-intercept is -3.

PROBLEMS

Find the x-intercept and the y-intercept of the graph

1. $-5x+7y=35$

2. $3x-2y=-6$

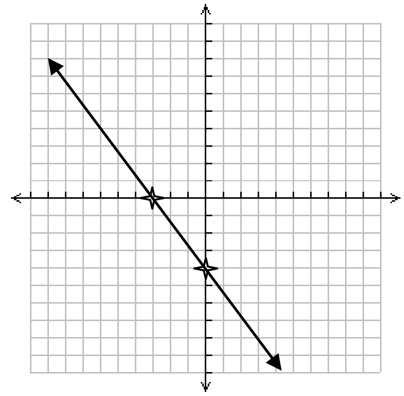
3. $y=\frac{1}{4}x+5$

Example 2 **Use a graph to find the intercepts**

Identify the x-intercept and the y-intercept of the graph

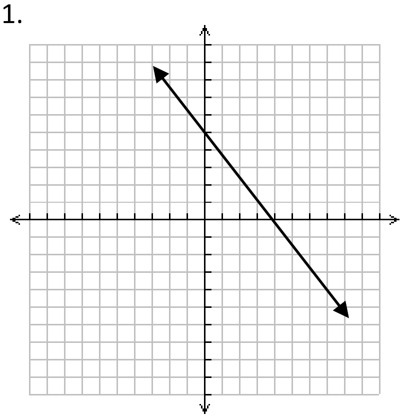
Solution: To find the x-intercept, identify the point where the line crosses the x-axis. **The x-intercept is -3.**

To find the y-intercept, identify the point where the line crosses the y-axis. **The y-intercept is -4**.



PROBLEMS: **Identify the x-intercept and the y-intercept of the graph**

1.



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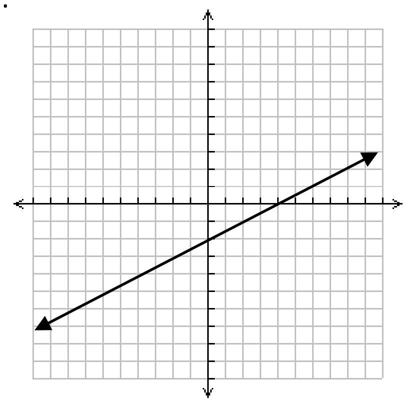

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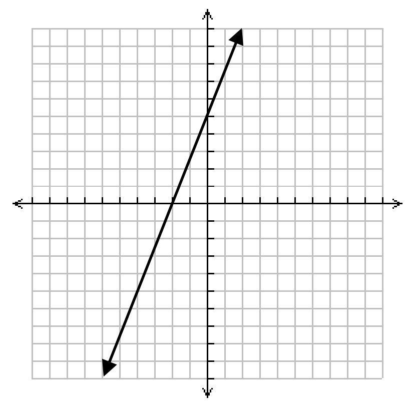

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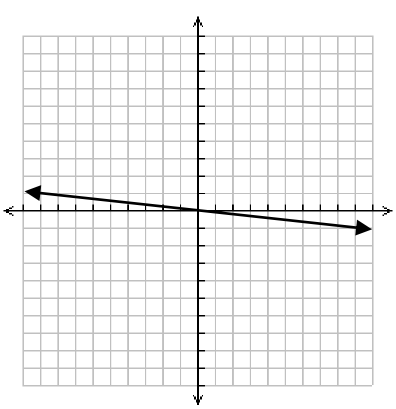

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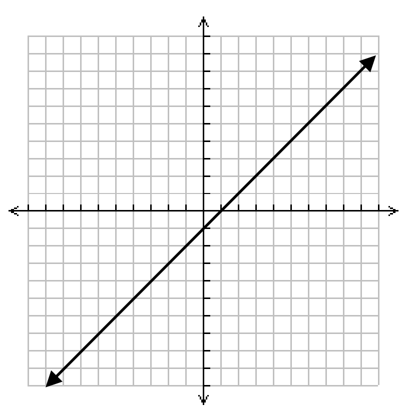
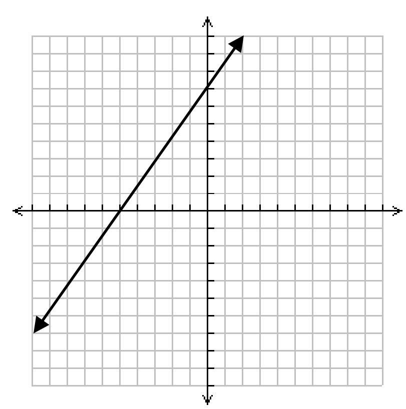
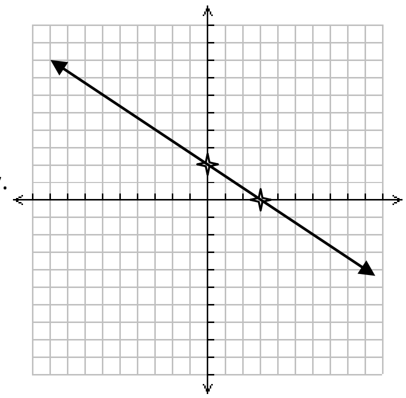


6.



Example 3 Use intercepts to graph an equation

Graph $2x + 3y = 6$. Label the points where the line crosses the axes.



Solution:

Step 1: Find the intercepts: To find the y-intercept, set $x = 0$ and solve for y.

$2(0) + 3y = 6$

Solve for $y = 2$

To find the x-intercept, set $y = 0$ and solve for x.

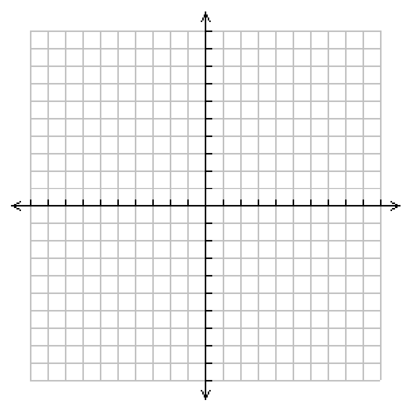
$2x + 3(0) = 6$

Solve for $x = 3$

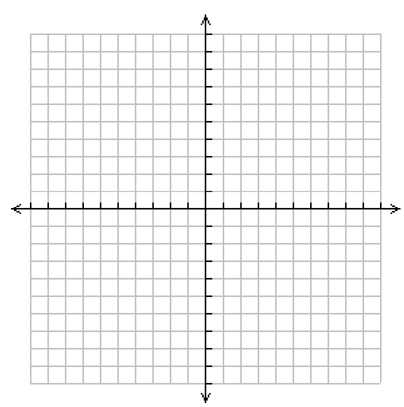
Step 2: Plot the points (intercepts) on the corresponding axes.

Step 3: Connect the points by drawing a line through them.

1. Graph $-3x + 2y = 6$. Label the points where the line crosses the axes.



2. Graph $5x - 4y = 20$. Label the points where the line crosses the axes.



Lesson 3.4 Find the Slope and Rate of Change

Vocabulary

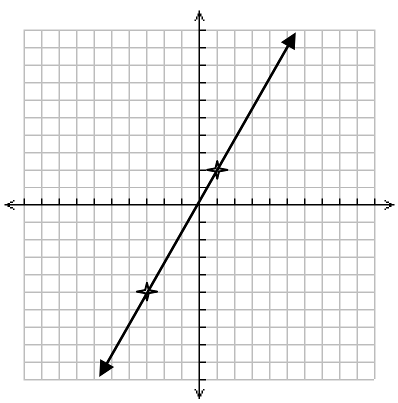
The **slope** of a line is the ratio of the vertical change (rise) to the horizontal change (run) between any two points of the line. **SLOPE = RISE/RUN**

Example 1 **Find a positive slope**

Find the slope of the line shown.

Solution

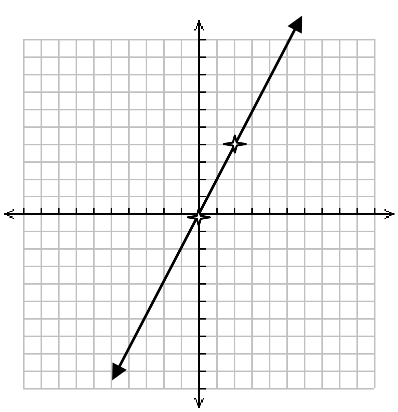
Pick two convenient points on the line (that fall on a lattice point). Count the boxes (units) you move up. Count the boxes (units) you move to the right. Divide the number of boxes (units) you moved up by the number of boxes (units) you moved to the right to get the slope.



Pick the points (1,2) and (-3,-5). You start at the lower point and move 7 boxes (units) up. Then you move 4 Boxes (units) to the right. The slope is $\frac{7}{4}$.

PROBLEMS: **Find the slope of the line shown.** **SLOPE = RISE/RUN**

1.



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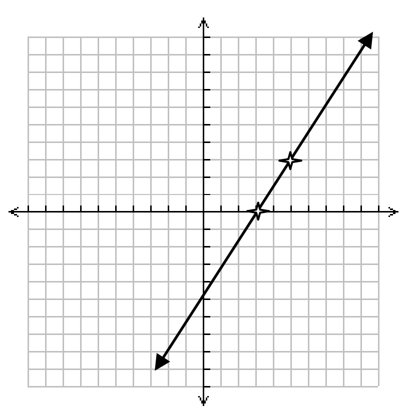

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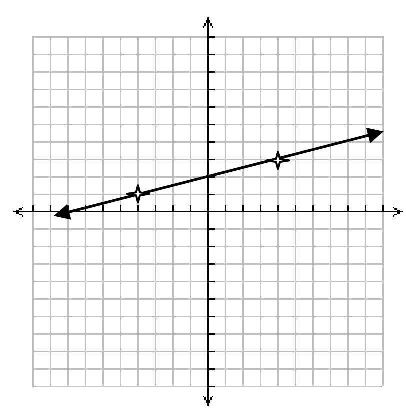
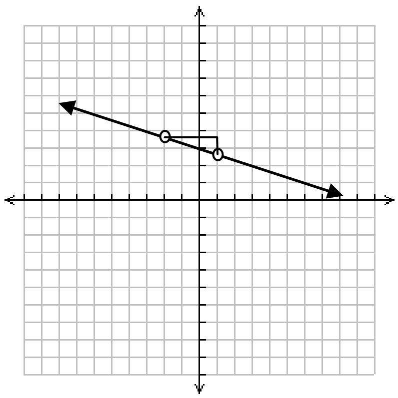


Example 2 **Find a negative slope**

Find the slope of the line shown.

Solution

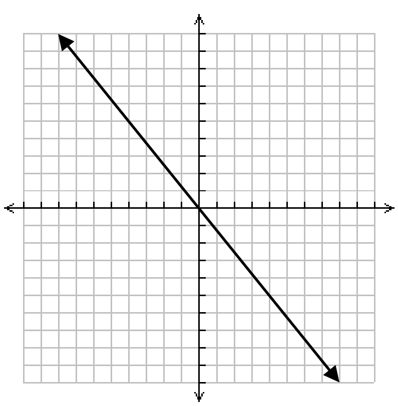
Pick two convenient points on the line (that fall on a lattice point). Count the boxes (units) you move up. Count the boxes (units) you move to the left. Divide the number of boxes (units) you moved up by the number of boxes (units) you moved to the left to get the slope.



Pick the points (1,3) and (-2,4). You start at the lower point and move 1 box (units) up. Then you move 3 Boxes (units) to the left. The slope is $-\frac{1}{3}$.

PROBLEMS: **Find the slope of the line shown.**

1.



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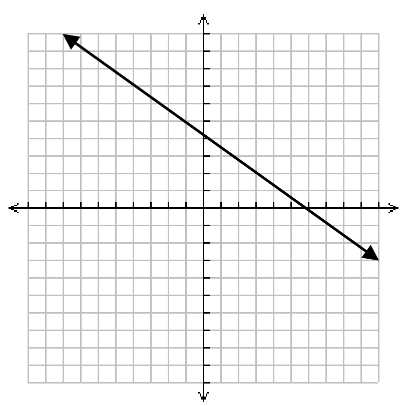

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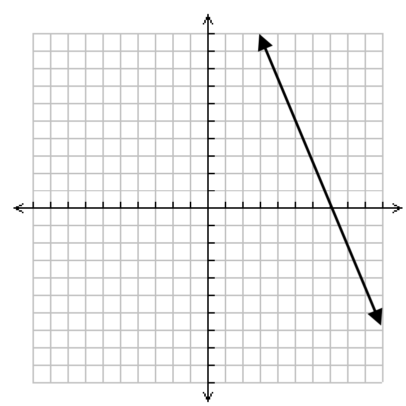


Lesson 3.5 Graph Using Slope-Intercept

Vocabulary

A linear equation (straight line relating x and y) of the form **y = mx + b** is written in **slope-intercept form**. The letter **m** stands for the slope (how steep the line is) and the letter **b** for the y-intercept (where the line crosses the y-axis).

Two **parallel** lines will not intersect each other and have the same slope.

Two lines are **perpendicular** to each other if they intersect each other at 90˚. Their slopes are negative reciprocals. For example if a line has a slope of 4 than a line perpendicular to that line will have a slope of -¼ (which is the negative reciprocal of 4).

Example 1 Identify the slope and the y-intercept

a. $y=\frac{1}{4}x+5$

Solution: The equation is in the form **y = mx + b.**

The slope is ¼. The y-intercept is 5.

b. $-5x+6y=12$

Solution: The equation is not in the form **y = mx + b.**

Rewrite the equation in slope-intercept form by solving for y.

$$-5x+6y=12$$

$$6y=5x+12 \qquad \textit{Add } 5x \textit{ to each side}$$

$$y=\frac{5}{6}x+2 \qquad \textit{Divide each side by } 6$$

The slope is 5/6. The y-intercept is 2.

PROBLEMS: Find the slope and y-intercept of the line shown.

1. y = 5x – 6

2. y = ½ x +3

3. y = -6x + 4

4. y = -8x

5. 2x + 3y = 6

6. y = 7 + 3x

Example 2: Graph the equation 2x + y = 3 using slope intercept form.

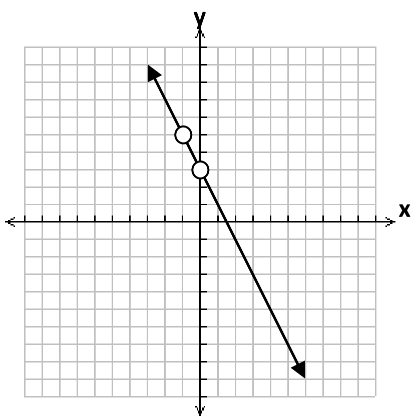
Solution:

Step 1: Rewrite the equation in slope-intercept form: y= -2x + 3

Step 2: Identify the slope and the y-intercept: slope: -2 y-intercept: 3

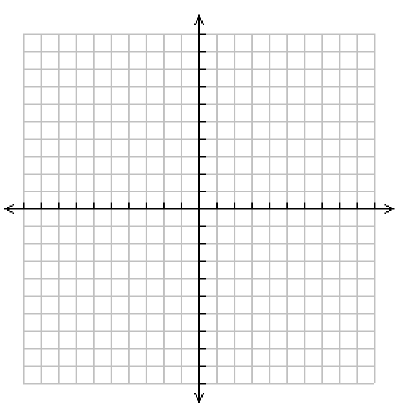
Step 3: Mark the point (0,3) as y-intercept on the y-axis.

Step 4: From the y-intercept draw a slope of -2. -2 can be rewritten as $-2 = \frac{2}{-1}$. Therefore, you rise 2 and run -1. That means from the y-intercept of 3 you go up 2 (in the positive y-direction) and then 1 to the left (in the negative x-direction).

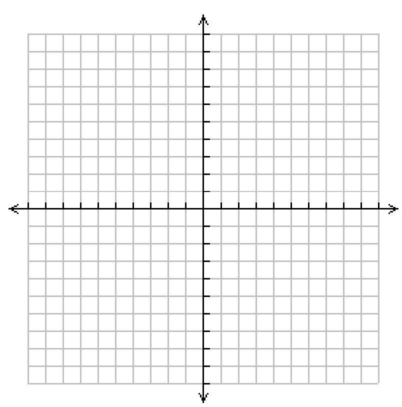


PROBLEMS: **Graph an equation using slope intercept form**

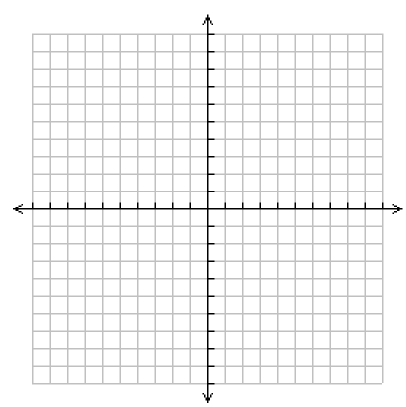
1. y = 5x – 6



2. y = ½ x +3



3. 2y + 6x = 4



Lesson 3.6 Predict with Linear Models

Vocabulary

A line that best represents a trend in data or points is called the **best-fitting line**.

Using a line or its equation to approximate values between two known points is called **linear interpolation**.

Using a line or its equation to approximate values outside the range of two known points is called **linear extrapolation**.

Example 1 **Find the equation of the best fitting line.**

You are given following data:

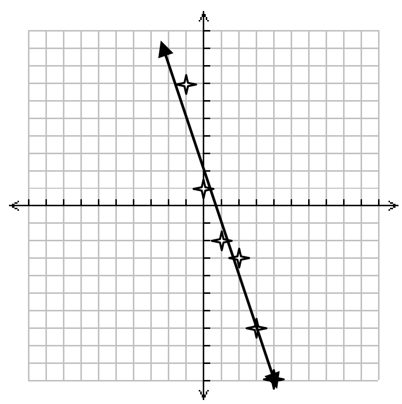
X	-1	0	1	2	3	4
y	7	1	-2	-3	-7	-10

Find the equation of the best-fitting line for the data.

Solution:

Step 1: Draw a **straight** line representing the points.

Step 2: Find the equation of the line.

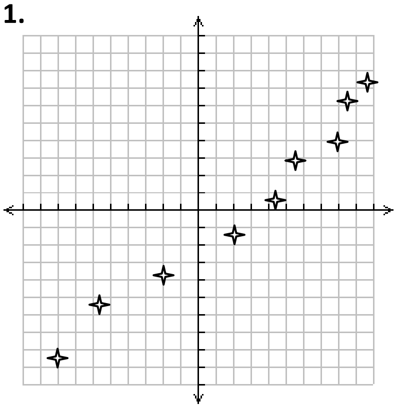


The y-intercept is 2 and the slope is -3.

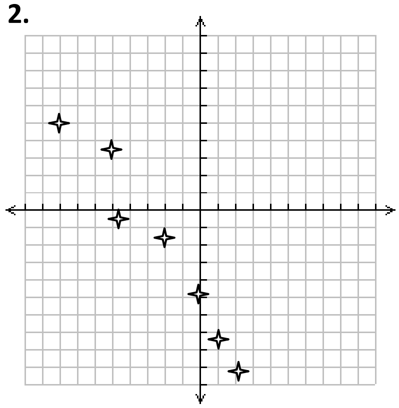
Therefore, the equation of the line is **y = -3x + 2**

PROBLEMS: Find the equation of the best fitting line.

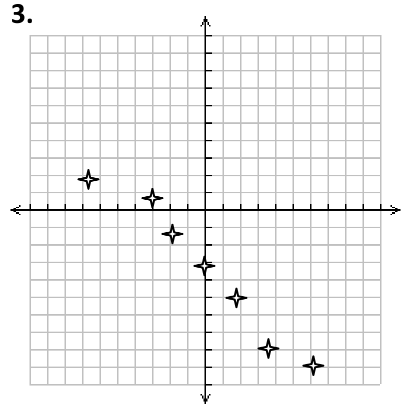
1.



2.

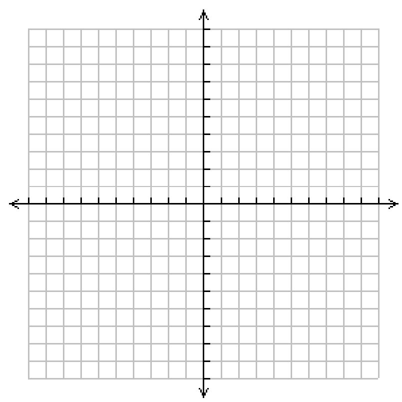


3.



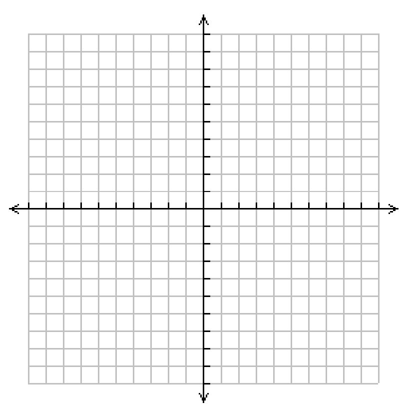
4. Make a scatter plot of the data. Then find the equation of the best fitting line.

x	-3	-2	0	1	3	4
y	-8	-4	-2	0	2	6



5. Make a scatter plot of the data. Then find the equation of the best fitting line.

x	-3	-2	0	2	3	4
y	3	2	-1	0	-1	-2



Lesson 3.7 Comparing Linear and Exponential Functions

Vocabulary

A linear function can be represented in a straight line in the form of $y = mx + b$.

A exponential function can be represented in a curved line and has the form $y = a(1+b)^x$.

Example 1 John deposits $300 into an account which earns 8% interest each year on the original deposit. Jane deposits $300 into an account which earns 8% interest each year on the year's end balance. How much money is in each account after 3 years?

Solution Firstly, 2% has to be converted to a decimal: $8\% = \frac{8}{100} = 0.08$

John's interest for each year is: $\$300 \times 0.08 = \$24$

Therefore, the table below shows the balance of John's account after each of the 3 years:

Jack's Account Balance		
Year 1	Year 2	Year 3
$324	$348	$372

Jane's interest for year 1 is just the same as John's: $\$300 \times 0.08 = \$24$. Therefore, Jane will have a balance of $324 on her account. However, for year 2, the interest is computed using the balance of $324: $\$324 \times 0.08 \approx \$26$. The interest of $26 is added to the balance of $324 to yield an account balance of $350 (324+26=350). Therefore, for year 3, the interest is computed using the balance of $350 is: $\$350 \times 0.08 = \$28$. Adding the interest of $28 to the account balance of $350 yields $378.

The table below shows the balance of Jane's account after each of the 3 years:

Jane's Account Balance		
Year 1	Year 2	Year 3
$324	$350	$378

While John gets an identical interest payment of $24 per year Jane's interest payment increases each year since she earns interest on the interest accumulated. John's interest can be explained as a **linear function** (same interest payment each year) while Jane's interest can be modeled with an **exponential function** (interest payment increases each year).

PROBLEMS

1. Abdul deposits $500 into an account which earns 12% interest each year on the original deposit. Max deposits $500 into an account which earns 12% interest each year on the year's end balance. How much money is in each account after 3 years? Show your answer in table format.

Abdul's Account Balance		
Year 1	Year 2	Year 3

Max' Account Balance		
Year 1	Year 2	Year 3

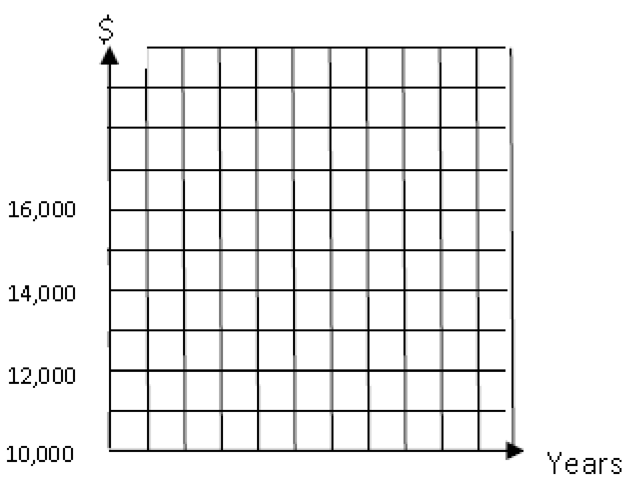
2. Ted deposits $10,000 into an account which earns 15% interest each year on the original deposit. Rick deposits $10,000 into an account which earns 15% interest each year on the year's end balance. How much money is in each account after 3 years? Show your answer in table format.

Ted's Account Balance		
Year 1	Year 2	Year 3

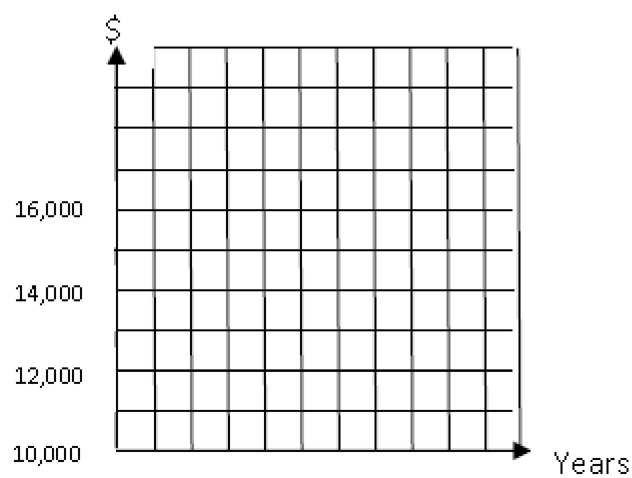
Rick's Account Balance		
Year 1	Year 2	Year 3

3. Plot the table for Ted and Rick: Years on the x-axis and interest earned (account balance – initial deposit) on the y-axis.

TED



RICK



Which one has a straight line and which one shows an upward curved line?

Which one is linear and which one is exponential?

4. To encourage communication between parents and their children and to prevent children from having extremely large monthly bills due to additional minute charges, two cell phone companies are offering special service plans for students.

Talk Fast cellular phone service charges $0.10 for each minute the phone is used.

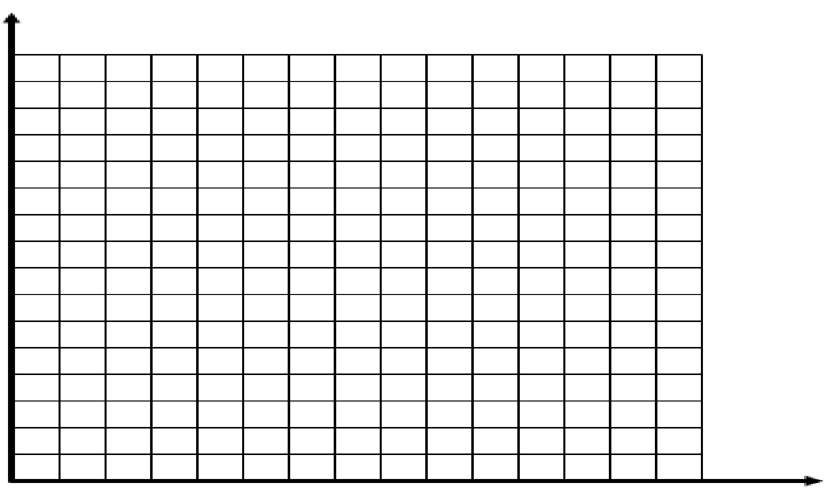
Talk Easy cellular phone service charges a basic monthly fee of $18 plus $0.04 for each minute the phone is used.

Your parents are willing to purchase for you one of the cellular phone service plans listed above. However, to help you become fiscally responsible they ask you to use the following questions to analyze the plans before choosing one.

a. How much would each company charge per month if you talked on the phone for 100 minutes in a month? How much if you talked for 200 minutes in a month?

b. Build a table and make a graph for Talk Fast:

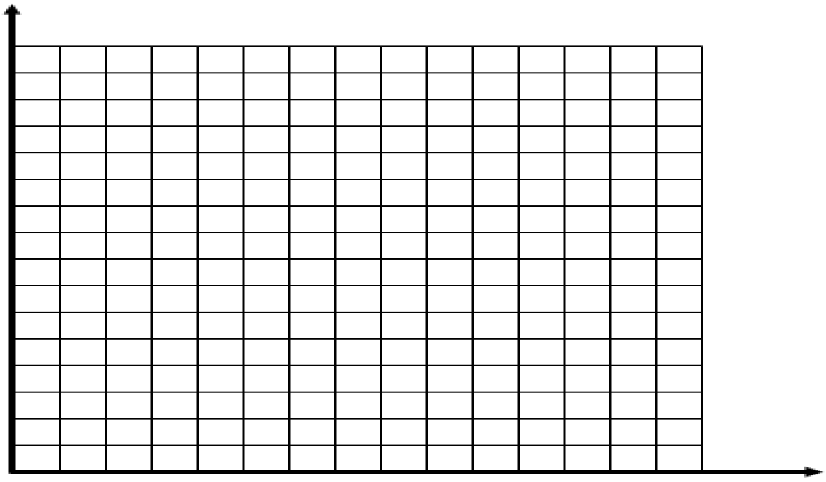
X (number of minutes)															
Y (cost in $)															



c. Write a function rule (in the form y=mx+b), where y represents the cost ($) and x represents the minutes.

d. Build a table and make a graph for Talk Easy:

X (number of minutes)															
Y (cost in $)															



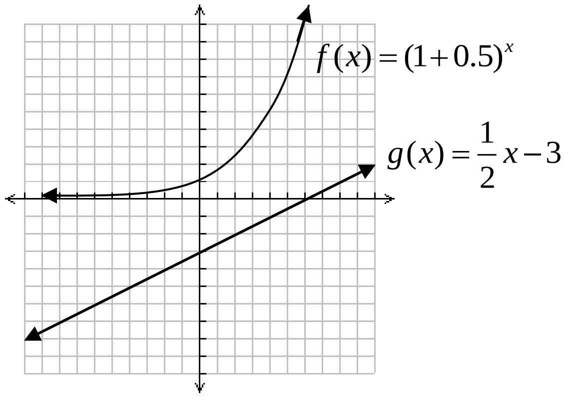
e. Write a function rule (in the form y= mx+b), where y represents the cost ($) and x represents the minutes.

f. Which company would be a better financial deal if you plan to use the phone for 200 minutes a month? Explain your reasoning.

g. Which company would be a better financial deal if you plan to use the phone for 500 minutes a month? Explain your reasoning.

5. Compare the graphs below.

a. Which one is shown as a straight line and which one is curved?



b. Show the graphs in table format.

	-3	-2	-1	0	1	2	3	4	5
$g(x) = \frac{1}{2}x - 3$									
$f(x) = (1+0.5)^x$									

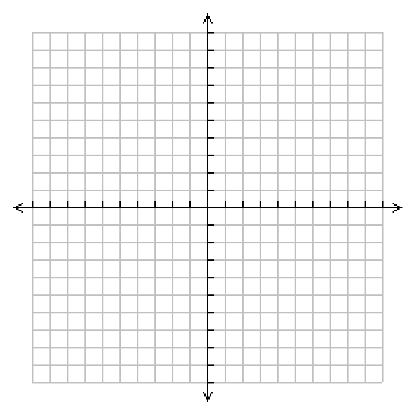
c. As the values of x increase by 1 what happens to the corresponding y-values of $g(x)$?

d. As the values of x increase by 1 what happens to the corresponding y-values of $f(x)$?

e. Which function increases in same increments? That function is called linear function.

f. Which function increases in "growing" increments? That function is called exponential function.

g. Write down an equation for a linear function (different from above) and plot the function.



h. Write down an equation for an exponential function (different from above) and plot the function.

Lesson 3.8 Explicit and Recursive Formulas

Vocabulary

An **explicit formula** allows direct computation of any term for a sequence $a_1, a_2, a_3, a_4, \ldots a_n, \ldots.$

$a_n = a_1 + (n-1) \cdot d$ where a_1 is the first term, and d is the difference between subsequent terms.

Example: $a_1 = 2, a_2 = 5, a_3 = 8, a_4 = 11, \ldots\ldots$ Therefore, $a_1 = 2$, and $d = 3$.

To find any term in the sequence plug the values for a_1 and d into the formula:

$$a_n = a_1 + (n-1) \cdot d$$
$$a_n = 2 + (n-1) \cdot 3$$

For a sequence $a_1, a_2, a_3, a_4, \ldots a_n, \ldots.$ a **recursive formula** is a formula that requires the computation of all previous terms in order to find the value of a_n.

Example:

$$a_1 = 3$$
$$a_n = 2a_{n-1} + 5$$

$$a_1 = 3$$
$$a_2 = 2a_{n-1} + 5 = 2a_{2-1} + 5 = 2a_1 + 5 = 2(3) + 5 = 11$$
$$a_3 = 2a_{n-1} + 5 = 2a_{3-1} + 5 = 2a_2 + 5 = 2(11) + 5 = 27$$
$$a_4 = 2a_{n-1} + 5 = 2a_{4-1} + 5 = 2a_3 + 5 = 2(27) + 5 = 59$$
$$etc.$$

Example 1 Write the terms of a sequence. Write the first five terms of

a. $a_n = 4n + 3$ b. $a_n = (-1)^{n+1}$

Solution: Because no domain is specified, start with n=1.

a.

$$a_1 = 4(1) + 3 = 7$$
$$a_2 = 4(2) + 3 = 11$$
$$a_3 = 4(3) + 3 = 15$$
$$a_4 = 4(4) + 3 = 19$$
$$a_5 = 4(5) + 3 = 23$$

b.

$$a_1 = (-1)^{1+1} = 1$$
$$a_2 = (-1)^{2+1} = -1$$
$$a_3 = (-1)^{3+1} = 1$$
$$a_4 = (-1)^{4+1} = -1$$
$$a_5 = (-1)^{5+1} = 1$$

PROBLEMS Write the first six terms of the sequence.

1. $a_n = n - 3$

2. $a_n = 4n - 5$

3. $a_n = 2^{n+1}$

4. $a_n = \frac{(-1)^n}{2n}$

Find the explicit formula for each sequence.

5. 3, 8, 13, 18, …..

6. -7, -5, -3, -1, …..

7. 12, 9, 6, 3, …..

8. 5, 11, 16, 21, …..

Find the recursive formula for each sequence.

9. 3, 8, 13, 18, …..

10. -7, -5, -3, -1, …..

11. 12, 9, 6, 3, …..

12. 5, 11, 16, 21, …..

9. The first term in this sequence is −1. Which function represents the sequence?

n	1	2	3	4	5	…
a_n	-1	1	3	5	7	…

A. $n + 1$
B. $n + 2$
C. $2n - 1$
D. $2n - 3$

10. Which function is modeled in this table?

x	**f(x)**
1	**8**
2	**11**
3	**14**
4	**17**

A. $f(x) = x + 7$
B. $f(x) = x + 9$
C. $f(x) = 2x + 5$
D. $f(x) = 3x + 5$

11. Which explicit formula describes the pattern in this table?

d	**C**
2	6.28
3	9.42
5	15.70
10	31.40

A. $d = 3.14 \times C$
B. $3.14 \times C = d$
C. $31.4 \times 10 = C$
D. $C = 3.14 \times d$

12. If $f(12) = 4(12) - 20$, which function gives $f(x)$?

A. $f(x) = 4x$
B. $f(x) = 12x$
C. $f(x) = 4x - 20$
D. $f(x) = 12x - 20$

Lesson 3.9 Arithmetic Sequences and Series

Goal: Study arithmetic sequences and series.

In an **arithmetic sequence**, the difference of consecutive terms is constant. That means the difference between any two consecutive terms remains the same. This difference is called **common difference** and is denoted by d in the formula below. The nth term of an arithmetic sequence with first term a_1 and common difference d is given by:

$$a_n = a_1 + (n-1)d$$

The expression to add the terms in an arithmetic sequence is called an **arithmetic series**. The sum of n terms is given by:

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

Example 1 Identify arithmetic sequences. Which ones of the following sequences is arithmetic?

a. $2,4,6,8,.....$ **Solution**: Yes, because the difference between all terms is 2.

b. $1,4,7,9,.....$ **Solution**: No, because the difference between all terms is not the same.

c. $5,9,13,17,.....$ **Solution**: Yes, because the difference between all terms is 4.

PROBLEMS Identify arithmetic sequences. Which ones of the following sequences are arithmetic?

1. $3,6,9,12,.....$

2. $4,8,12,16,..$

3. $8,6,4,2,-2,...$

4. $1,3,5,7,11,13,...$

Example 2	Write a rule for the nth term of the sequence. Then find a_{15}.
	$19, 23, 27, 31, 35, \ldots$
Solution:	The difference between each term is 4. Therefore $d = 4$.
	The first term is 19. Therefore, $a_1 = 19$.
	Plug the values into the formula $\quad a_n = a_1 + (n-1)d = 19 + (n-1)\cdot 4$.
	Therefore, the rule for the nth term is: $\quad a_n = 19 + (n-1)\cdot 4$.
	To find the 15th term or a_{15} replace n with 15: $\quad a_{15} = 19 + (15-1)\cdot 4 = 19 + 14\cdot 4 = 75$.

PROBLEMS Write a rule for the nth term of the sequence. Then find a_{15}.

5. $5, 7, 9, 11, 13, \ldots$

6. $6, 11, 16, 21, \ldots$

7. $4, 3, 2, 1, \ldots$

8. $-4, 2, 8, 14, 20, \ldots$

9. $-25, -29, -33, -37, \ldots$

Example 3	Find the sum of the first 25 terms of the arithmetic series $19, 23, 27, 31, 35, \ldots$.
Solution:	The difference between each term is 4. Therefore $d = 4$.
	The first term is 19. Therefore, $a_1 = 19$.
	Plug the values into the formula $a_n = a_1 + (n-1)d = 19 + (n-1)\cdot 4.$
	Therefore, the rule for the nth term is: $a_n = 19 + (n-1)\cdot 4.$
	To find the 25th term or a_{25} replace n with 25: $a_{25} = 19 + (25-1)\cdot 4 = 19 + 24\cdot 4 = 115.$
	Plug the values for a_1, a_{25}, d into the formula $S_n = n\left(\frac{a_1 + a_n}{2}\right)$
	or $= S_{25} = 25\left(\frac{19+115}{2}\right) = 1675$

PROBLEMS Find the sum of the first n terms of the arithmetic series.

10. $5, 7, 9, 11, 13, \ldots;\ n = 19$

11. $25, 35, 45, 55, \ldots;\ n = 50$

12. $4, 3, 2, 1, \ldots;\quad n = 30$

13. $5, 8, 11, 14, \ldots;\quad n = 10$

Example: Find an explicit and recursive formula for the sequence $5, 8, 11, 14, 17, ...$

Solution:

An **explicit formula** can be written in the form $a_n = a_1 + (n-1) \cdot d$ where a_1 is the first term, and d is the difference between subsequent terms.

The first term, $a_1 = 5$. The difference between each term, $d = 3$.

To find any term in the sequence plug the values for a_1 and d into the formula:

$$a_n = a_1 + (n-1) \cdot d$$
$$a_n = 5 + (n-1) \cdot 3$$

To check whether the explicit formula for the sequence is correct check by putting in the values for a_1 and d for let's say the 4th term: $a_4 = a_1 + (n-1) \cdot d = 5 + (4-1)3 = 5 + (3)3 = 14$

This is indeed correct.

A **recursive formula** is a formula that requires the computation of all previous terms in order to find the value of a_n. In our case $a_1 = 5$. The next term can be written as $a_n = a_{n-1} + 3$.

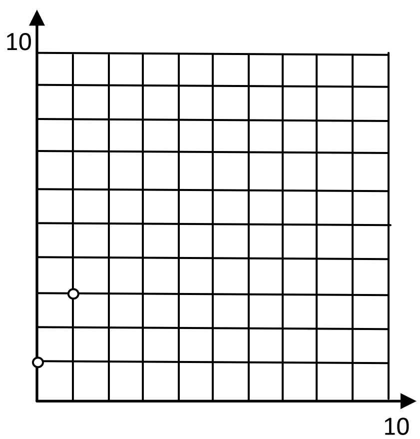
To check whether this is correct put in the values for the first 4 terms:

$$a_1 = 5$$
$$a_2 = a_{n-1} + 3 = a_{2-1} + 3 = a_1 + 3 = 5 + 3 = 8$$
$$a_3 = a_{n-1} + 3 = a_{3-1} + 3 = a_2 + 3 = 8 + 3 = 11$$
$$a_4 = a_{n-1} + 3 = a_{4-1} + 3 = a_3 + 3 = 11 + 3 = 14$$
$$etc.$$

SEQUENCES ARE FUNCTIONS

14. Represent the function $f(n) = 2n+1$ in table form and graphically. Fill in the remaining points.

	0	1	2	3	4	5
$f(n) = 2n+1$	1	3				

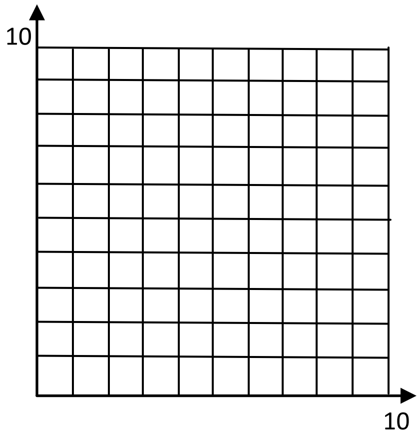


15. Write an explicit formula for the function in the form $a_n = a_1 + (n-1)d$ where a_1 is the first term and d is the difference between two consecutive terms.

16. Write a recursive formula for the function. (Example : $\begin{matrix} a_1 = 3 \\ a_n = a_{n-1} + 5 \end{matrix}$. The next term is expressed in terms of the previous term).

17. Represent the function $f(n)=n+3$ in table form and graphically.

	0	1	2	3	4	5
$f(n)=n+3$						



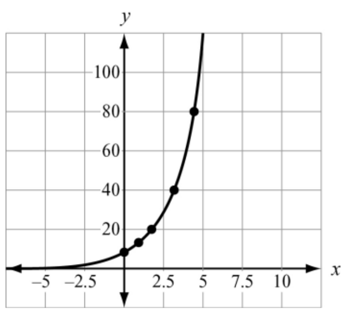
18. Write an explicit formula for the function in the form $a_n = a_1 + (n-1)d$ where a_1 is the first term and d is the difference between two consecutive terms.

19. Write a recursive formula for the function. (Example : $\begin{matrix} a_1 = 3 \\ a_n = a_{n-1} + 5 \end{matrix}$ The next term is expressed in terms of the previous term).

20. A farmer owns a horse that can continuously run an average of 8 miles an hour for up to 6 hours. Let *y* be the distance the horse can travel for a given *x* amount of time in hours. The horse's progress can be modeled by a function. Which of the following describes the domain of the function?

A. $0 \leq x \leq 6$
B. $0 \leq y \leq 6$
C. $0 \leq x \leq 48$
D. $0 \leq y \leq 48$

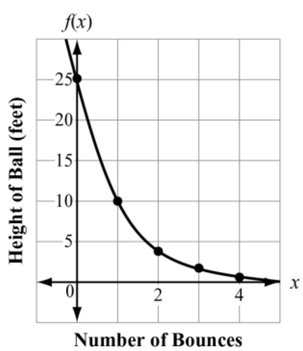
21. A population of squirrels doubles every year. Initially there were 5 squirrels. A biologist studying the squirrels created a function to model their population growth, $P(t) = 5(2t)$ where t is time. The graph of the function is shown.



What is the range of the function?

A. any real number
B. any whole number greater than 0
C. any whole number greater than 5
D. any whole number greater than or equal to 5

22. The function graphed on this coordinate grid shows *y*, the height of a dropped ball in feet after its *x*th bounce.

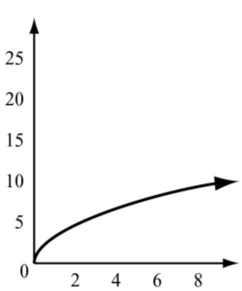


On which bounce was the height of the ball 10 feet?

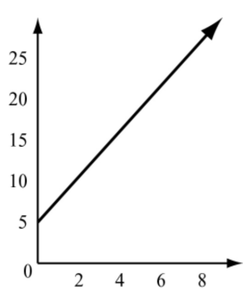
A. bounce 1
B. bounce 2
C. bounce 3
D. bounce 4

23. To rent a canoe, the cost is \$3 for the oars and life preserver, plus \$5 an hour for the canoe. Which graph models the cost of renting a canoe?

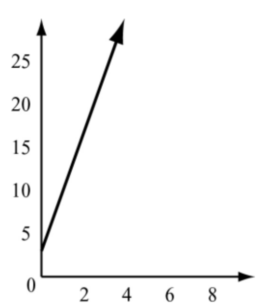
A.



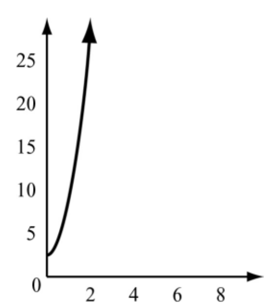
B.



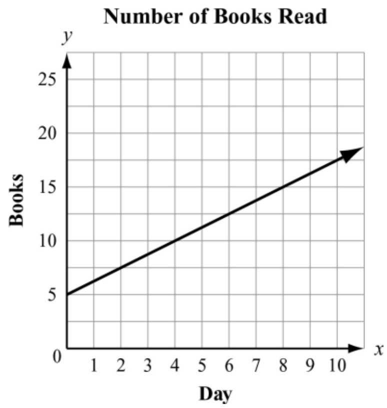
C.



D.



24. Juan and Patti decided to see who could read the most books in a month. They began to keep track after Patti had already read 5 books that month. This graph shows the number of books Patti read for the next 10 days.



If Juan has read no books before the fourth day of the month and he reads at the same rate as Patti, how many books will he have read by day 12?

A. 5
B. 10
C. 15
D. 20

25. A function *g* is an odd function. If *g*(⁻3) = 4, which of the points lie on the graph of *g*?

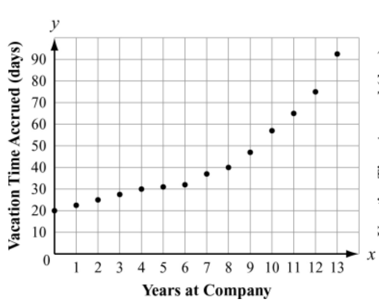
A. (3, ⁻4)
B. (⁻3, ⁻4)
C. (4, ⁻3)
D. (⁻4, 3)

26. Which statement is true about the function *f* (*x*)=7?

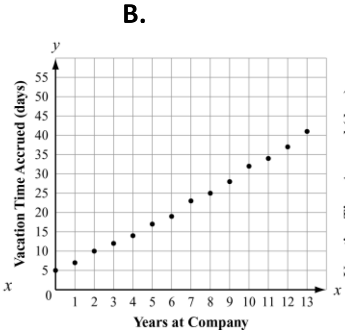
A. The function is odd because ⁻*f*(*x*) = ⁻*f*(*x*).
B. The function is even because ⁻*f*(*x*) = *f*(⁻*x*).
C. The function is odd because *f*(*x*) = *f*(⁻*x*).
D. The function is even because *f*(*x*) = *f*(⁻*x*).

27. Which scatter plot represents a model of linear growth?

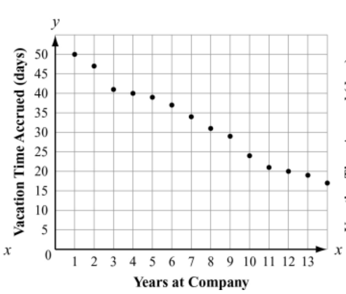
A.



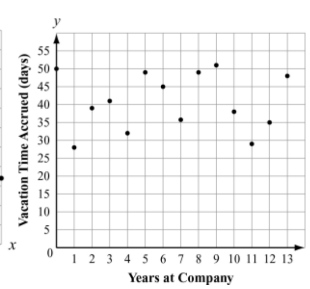
B.



C.

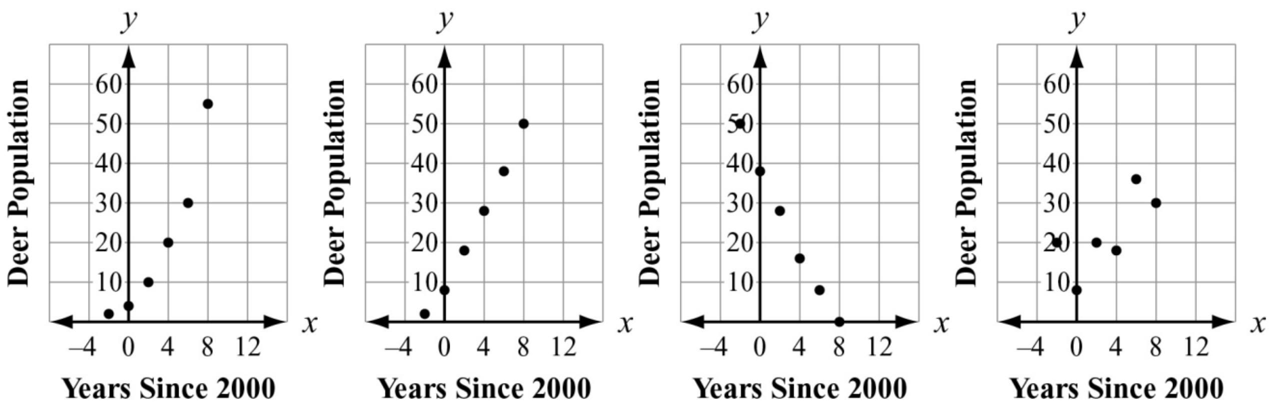


D.



28. Which scatter plot best represents a model of exponential growth?

A. **B.** **C.** **D.**



29. Which table represents a function with a variable growth rate?

A.

x	0	1	2	3	4
y	5	6	7	8	9

B.

x	0	1	2	3	4
y	0	22	44	66	88

C.

x	0	1	2	3	4
y	5	13	21	29	37

D.

x	0	1	2	3	4
y	0	3	9	27	81

30. If the parent function is $f(x) = mx + b$, what is the value of the parameter m for the curve passing through the points $(-2, 7)$ and $(4, 3)$?

A. -9

B. $-\frac{3}{2}$

C. -2

D. $-\frac{2}{3}$