Reasoning with Equations and Quantities **Name: _________________**

Date: ________________

Understand solving equations as a process of reasoning and explain the reasoning
MCC9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. *(Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses.)*
Solve equations and inequalities in one variable
MCC9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.*(Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.)*
Solve systems of equations
MCC9-12.A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. *(Limit to linear systems.)*
MCC9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
Represent and solve equations and inequalities graphically
MCC9-12.A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

LESSON 2.0 ALGEBRA EXERCISES

Solve for x

1. $5x+10=35$

2. $30-3x=0$

3. $0.5x-6.7=13.3$

4. $2x+10=4x-8$

5. $\frac{x}{4}=5$

6. $\frac{10}{x}=20$

7. $-10=-15+a$

8. $x+14=16$

9. $0 = 5x + 5x$

10. $138 = -6(2n - 7)$

11. $21 = 2(1 + 2x) - 5(x - 5)$

12. $-7 - 7(2m - 7) = 7(6 + m)$

13. $0.5x - 6.7 = 0.2x$

14. $7x + 10 = 4x - 8$

15. $\frac{2x}{4} = 5$

16. $\frac{10}{3x} = 30$

Solve each by factoring

17. $(x - 1)(x + 2) = 0$

16. $(2x + 4)(x + 5) = 0$

18. $x^2 + 4x + 3 = 0$

19. $x^2 - 10x + 21 = 0$

20. $x^2 + 5x - 24 = 0$

21. $x^2 - 7x - 8 = 0$

22. $x^2 - 15x + 56 = 0$

23. $x^2 + 7x + 12 = 0$

Lesson 2.1 Add and Subtract Rational Expressions

Example 1 **Find the sum or the difference.**

a. $\frac{3}{7x}+\frac{11}{7x}=\frac{14}{7x}=\frac{2}{x}$

b. $\frac{3}{5-x}-\frac{3x-11}{5-x}=\frac{3-(3x-11)}{5-x}=\frac{-3x+14}{5-x}$

c. $\frac{3}{5}-\frac{3x}{3}=\frac{3\cdot 3}{5\cdot 3}-\frac{3x\cdot 5}{3\cdot 5}=\frac{9}{15}-\frac{15x}{15}=\frac{9-15x}{15}=\frac{3-5x}{5}$

d. $\frac{3}{x+3}+\frac{3x}{x+2}=\frac{3(x+2)}{(x+3)(x+2)}+\frac{3x(x+3)}{(x+2)(x+3)}=\frac{3(x+2)+3x(x+3)}{(x+2)(x+3)}$

$=\frac{3x+6+3x^2+9x}{(x+2)(x+3)}=\frac{3x^2+12x+6}{(x+2)(x+3)}$

PROBLEMS:

1. $\frac{3+x}{5x}+\frac{2}{5x}=$

2. $\frac{5+x}{4x}-\frac{3}{5x}=$

3. $\frac{3}{x+1}-\frac{3x-7}{x+1}=$

4. $\frac{3}{x+2}-\frac{2x}{x+1}=$

5. $\frac{3}{2x}-\frac{2x}{x}=$

6. $\frac{3}{2}-\frac{2x}{3}=$

7. $\frac{1}{4}+\frac{x}{6}=$

Lesson 2.2 Solve Rational Equations

Example 1 **Solve.**

$$\frac{6}{x+5}=\frac{x}{6}$$

Solution:

$\frac{6}{x+5}=\frac{x}{6}$	Write the equation
$6\cdot 6=x(x+5)$	Cross-multiply
$36=x^2+5x$	Multiply
$0=x^2+5x-36$	Bring all the terms on one side
$0=(x+9)(x-4)$	Factor
$x=-9$ or $x=4$	Solve for x

PROBLEMS:

1. $\frac{x}{27}=\frac{3}{x}$

2. $\frac{3}{x}=\frac{2}{x+4}$

3. $\frac{x}{1}=\frac{-12}{x+7}$

4. $\frac{x}{1}=\frac{15}{x-2}$

5. $\frac{x}{2}+\frac{x}{3}=2$

1. Emily wants to solve the equation $ax - w = 3$ for *w*. Which equation shows the results of a correctly applied strategy?

A. $w = ax - 3$
B. $w = ax + 3$
C. $w = 3 - ax$
D. $w = 3 + ax$

2. Which equation is equivalent to $\frac{7x}{4} - \frac{3x}{8} = 11$?

A. $17x = 88$
B. $11x = 88$
C. $4x = 44$
D. $2x = 44$

3. Which equation is equivalent to $4n=2(t-3)$ when solved for *t*?

A. $t = \frac{4n-2}{3}$
B. $t = \frac{4n-3}{2}$
C. $t = \frac{4n+6}{2}$
D. $t = 4n-3$

Lesson 2.3 Solve a Linear System by Graphing

Goal Solve systems of linear equations

Vocabulary A **system of linear equations** with two variables x and y consists of two equations that can be written in the following form: $Ax + By = C \quad and \quad Dx + Ey = F.$

A **solution of a system** of linear equations with two variables is an ordered pair (x, y) that satisfies each equation (plugging in the corresponding numbers for $x \, and \, y$).

A system which has at least one solution is called **consistent**. The solution point is where both lines cross which was defined earlier as an ordered pair.

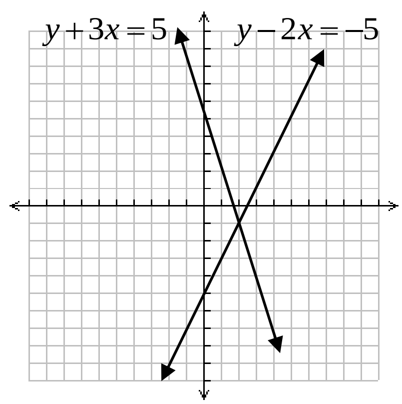
If there is no solution to a system, it is called **inconsistent** (if both lines are parallel they never touch; therefore there is no solution to the system).

A consistent system with exactly one solution is called **independent**.

A consistent system with infinitely many solutions is called **dependent**: Both lines lay on top of each other and every point is a solution.

Example 1: **Solve systems graphically**

$$y + 3x = 5 \qquad Equation\,1$$
$$y - 2x = -5 \qquad Equation\,2$$



Solution: Graph both equations. The lines intersect each other at $(2, -1)$, i.e. ordered pair.

Check the solution by plugging in the ordered pair into the equations:

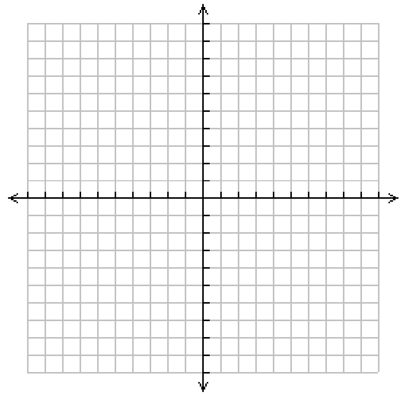
Equation 1	Equation 2
$y + 3x = 5$	$y - 2x = -5$
$(-1) + 3(2) = 5$	$(-1) - 2(2) = -5$
$-1 + 6 = 5$	$-1 - 4 = -5$
$5 = 5$	$-5 = -5$

PROBLEMS

Graph the linear system and find its solution. Then check the solution algebraically.

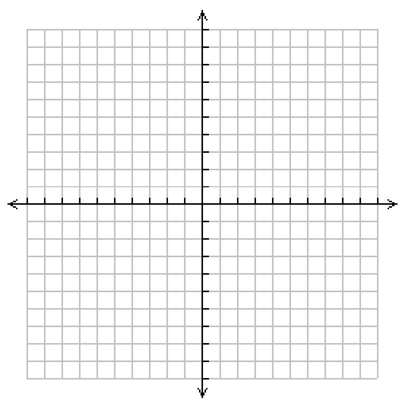
1. $y = 4x - 1$

$y = -x + 4$



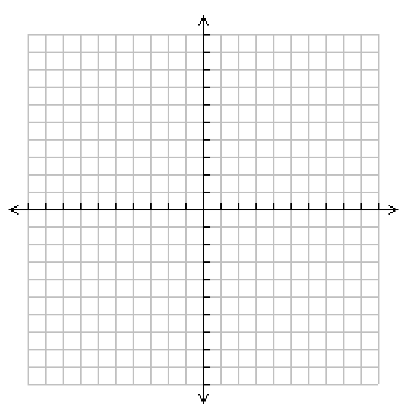
2. $3x + y = 7$

$y = 2x - 3$



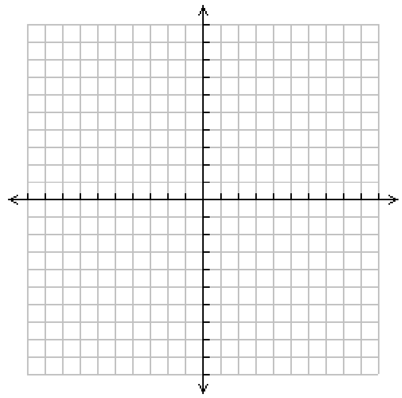
3. $2x + 2y = 4$

$y = 1$



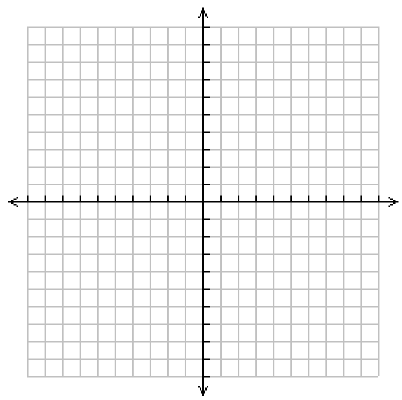
4. $y = -2x$

$y = x + 3$



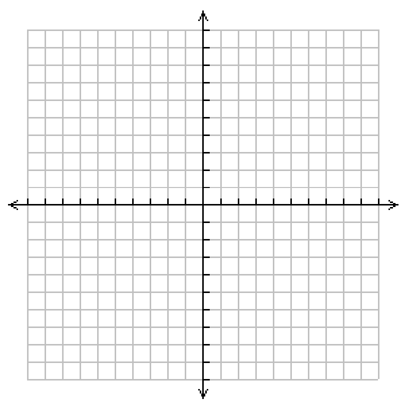
5. $y = x + 2$

$y = -\frac{1}{2}x + 5$

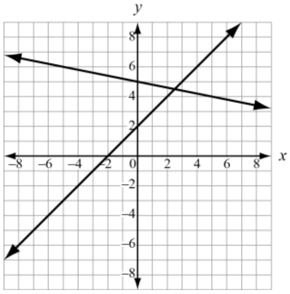


6. $y = x$

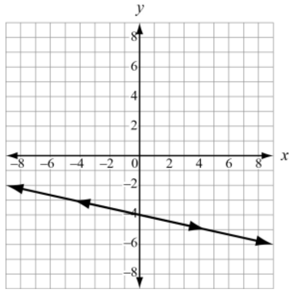
$y = 3$



7. Which graph would represent a system of linear equations that has multiple common coordinate pairs?



A



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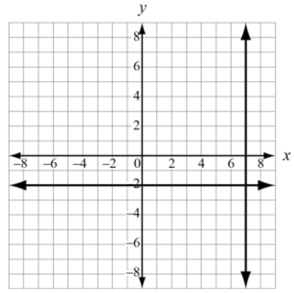

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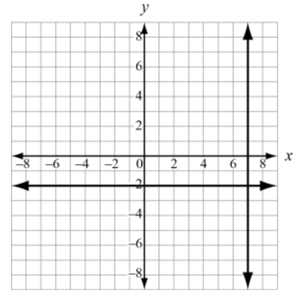


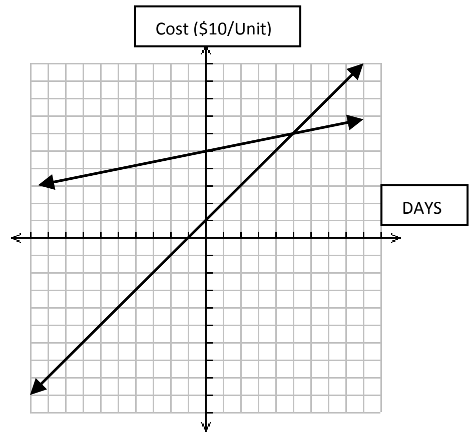
D

Example 3: **Use a linear system to solve a realistic problem.**

Car rental company A charges \$10 per day plus a one-time \$10 rental fee. Car rental company B charges \$2 per day plus a one-time \$50 rental fee. After how many days will the total cost of both options be the same?

Solution: Let x represent the number of days you rent the car. Let y be the total rental cost.

$$y = 10x + 10 \qquad Equation\,1(Car\ rental\ company\ A)$$
$$y = 2x + 50 \qquad Equation\,2(Car\ rental\ company\ B)$$



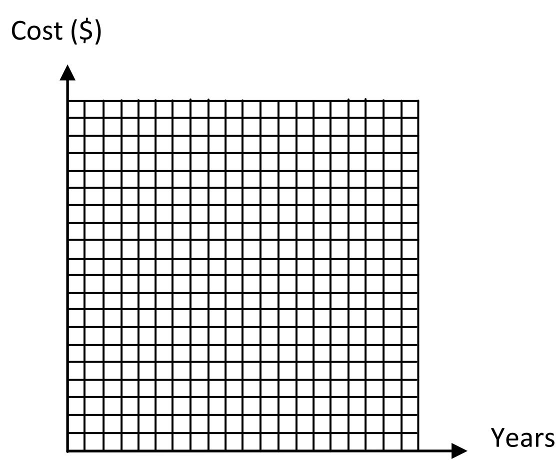
To solve the system graph the equations. The lines intersect at (5,60). Thus after 5 days the total cost (\$60) is the same at both rental car companies.

PROBLEMS

8. The price of car A is \$15000, and the price of car B is \$20000. Car A has yearly operating costs of \$3000, while car B has yearly operating costs of \$2000.

a. Write an equation for the cost of owning car A and an equation for the cost of owning car B.

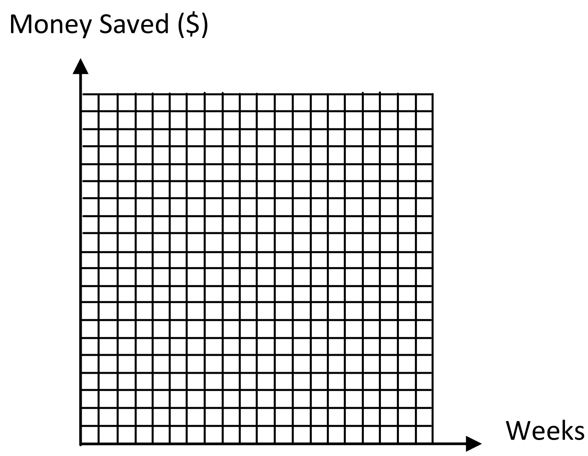
b. Graph both equations from part a. After how many years are the total costs of owning the cars equal?



c. State a reason why you would prefer to own one car over the other.

9. John decides to open a bank account. He deposits \$35 and plans to deposit every week another \$10. Markus has saved \$60 and plans to save \$5 every week.

a. Plot a line which represent John's money, and plot a line that represents Markus' money over time.



b. When do both lines meet? What does that point mean?

c. Who will have more money after 3 weeks?

d. Who will have more money after 8 weeks?

e. Why are the slopes of both lines different from each other?

f. What does that the slope of each line represent?

g. What do the y-intercepts of each line represent?

10. This equation can be used to find *h*, the number of hours it takes Flo and Bryan to mow their lawn: $\frac{h}{3}+\frac{h}{6}=1$

How many hours will it take them?

A. 6

B. 3

C. 2

D. 1

17. A ferry boat carries passengers back and forth between two communities on the Peachville River.

• It takes 30 minutes longer for the ferry to make the trip upstream than downstream.

• The ferry's average speed in still water is 15 miles per hour.

• The river's current is usually 5 miles per hour.

This equation can be used to determine how many miles apart the two communities are: $\frac{m}{15-5}=\frac{m}{15+5}+0.5$

What is *m,* the distance between communities?

A. 0.5 miles

B. 5 miles

C. 10 miles

D. 15 miles

18. Which expression represents all values of *x* for which the inequality $\frac{2}{3}+\frac{x}{3}>1$ is true?

A. $x < 1$

B. $x > 1$

C. $x < 5$

D. $x > 5$

Lesson 2.4 Solve Linear System Algebraically

Vocabulary	**Substitution Method**: See example 1 **Elimination Method**: See example 2.

Example 1 Solve the system using the **substitution method.**

$$\begin{aligned} 6x+3y&=12 \quad \textit{Equation}1 \\ 3x+y&=5 \quad \textit{Equation}2 \end{aligned}$$

Solution **Step 1:** Solve equation 2 for y:

$$y=-3x+5 \quad \textit{Equation}2$$

Step 2: Substitute the expression for y into Equation 1 and solve for x:

$$\begin{aligned} 6x+3(-3x+5)&=12 \quad \textit{Equation}1 \\ 6x-9x+15&=12 \\ -3x+15&=12 \\ -3x&=-3 \\ x&=1 \end{aligned}$$

Step 3: Substitute the value of x into either equation (we pick Equation 2) and solve for y.

$$\begin{aligned} 3(1)+y&=5 \quad \textit{Equation}2 \\ y&=2 \end{aligned}$$

The solution is (1,2).

<u>PROBLEMS</u> Solve the system using the substitution method

1. $\begin{aligned} 2x+y&=4 \\ 3x-5y&=6 \end{aligned}$

2. $\begin{aligned} 3x+6y&=3 \\ x-2y&=5 \end{aligned}$

Example 2 Solve the system using the **elimination method.**

$$\begin{aligned} 6x + 3y &= 12 \quad \textit{Equation}\,1 \\ 3x + y &= 5 \quad \textit{Equation}\,2 \end{aligned}$$

Solution **Step 1:** Multiply equation 2 by -3, so that the coefficients of y only differ in sign. Then add both equations and solve for x (since 3y is cancelled out by -3y):

$$\begin{array}{lll} 6x+3y=12 & & \textit{Equation}\,1 \rightarrow \\ 3x+y=5 & \bullet(-3) & \textit{Equation}\,2 \rightarrow \end{array} \qquad \begin{array}{r} 6x+3y=12 \\ \underline{-9x-3y=-15} \\ -3x \qquad =-3 \\ x=1 \end{array}$$

Step 2: Substitute the expression for y into Equation 1 and solve for x:

$$\begin{aligned} 6x+3y&=12 \quad \textit{Equation}\,1 \\ 6(1)+3y&=12 \\ 3y&=6 \\ y&=2 \end{aligned}$$

The solution is (1,2).

Hint: Alternatively you could have multiplied Equation 2 by -2 and cancelled x upon addition of both equations. Choose any way which is most convenient.

PROBLEMS Solve the system using the elimination method

3. $\begin{aligned} -3x+3y&=3 \\ 3x+y&=9 \end{aligned}$

4. $\begin{aligned} 4x-2y&=-2 \\ 6x+y&=5 \end{aligned}$

Solve the system using any method of your liking.

5. $\begin{aligned} 5x+7y&=-2 \\ 2x-7y&=9 \end{aligned}$

6. $\begin{aligned} 7x-3y&=6 \\ -2x+5y&=-10 \end{aligned}$

7. A manager is comparing the cost of buying ball caps with the company emblem from two different companies.
- company X charges a $50 fee plus $7 per cap
- company Y charges a $30 fee plus $9 per cap

For what number of ball caps will the manager's cost be the same for both companies?

A. 10 caps
B. 20 caps
C. 40 caps
D. 100 caps

8. A shop sells one-pound bags of peanuts for $2 and three-pound bags of peanuts for $5. If 9 bags are purchased for a total cost of $36, how many three-pound bags were purchased?

A. 3
B. 6
C. 9
D. 18

9. A restaurant has two specials.
Special 1: 2 hamburgers and 2 drinks for $18.
Special 2: 3 hamburgers and 1 drink for $17.

How much is a hamburger and how much is a drink?

A. $4, $5
B. $5, $5
C. $5, $4
D. $6, $5

Lesson 2.5 Graph Linear Inequalities in Two Variables

Example 1 Tell whether the ordered pairs $(4,-1)$ and $(-2,6)$ are solutions of the inequality $2x+3y>9$.

Solution: Substitute both points into the inequality and check whether they satisfy the inequality.

$$2(4)+3(-1)>9 \qquad\qquad 2(-2)+3(6)>9$$
$$5>9 \qquad\qquad 14>9$$

Not True **True**

Therefore, $(4,-1)$ is not a solution but $(-2,6)$ is a solution.

PROBLEMS Tell whether the given ordered pairs are solutions of the inequality.

1. $4x-y\le 2$ $(2,4)$ $(1,-3)$

2. $2x+y<-3$ $(0,1)$ $(-3,1)$

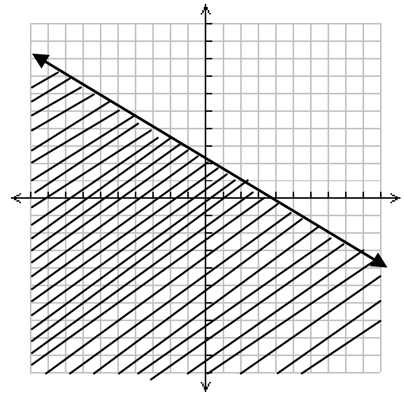
3. $2x+3y\le -3$ $(0,-1)$ $(-3,2)$

4. $-4x+2y\le -2$ $(1,2)$ $(3,4)$

Example 2 **Graph the linear equation** $2x+3y\le 6$.

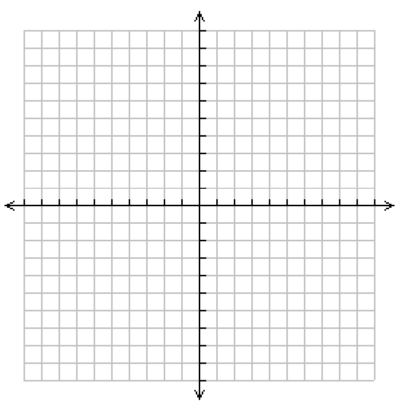
Solution: Solve the equation for $y=-\frac{2}{3}+2$. Then graph the line.

Plug in a point, say $(0,1)$. Since the point satisfies the inequality, the region containing the point is the solution and is shaded.

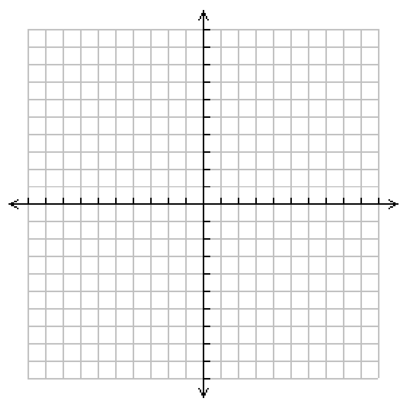


PROBLEMS: **Graph the linear equation**

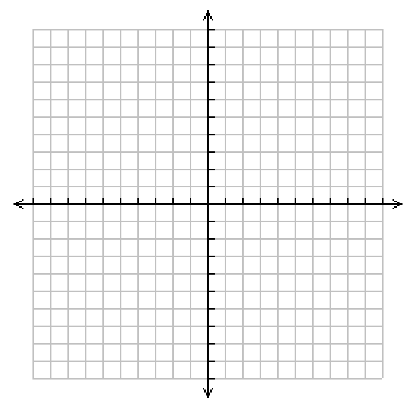
1. $2x+2y\ge -6$



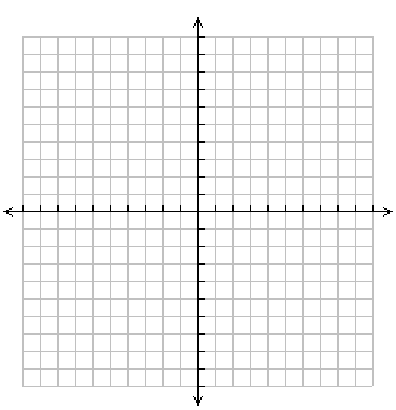
2. $-4x+2y>4$



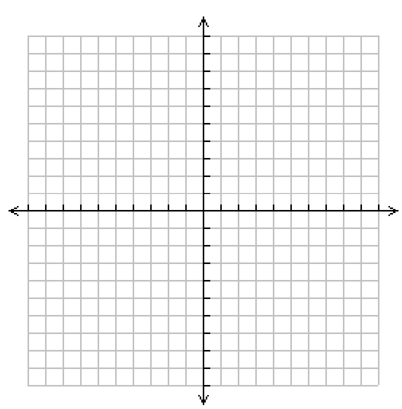
3. $x+y<3$



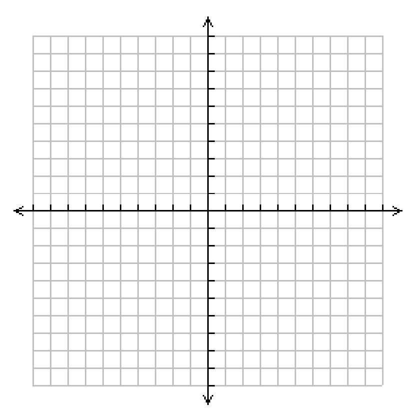
4. $y\ge -6$



5. $x<2$



6. $4x+y\ge 2$



Example 3 **In basketball you can score by throwing two-point and three-point shots.**

a. Write an inequality to represent the number of two-point shots and the number of three-point shots that total at least 15 points.

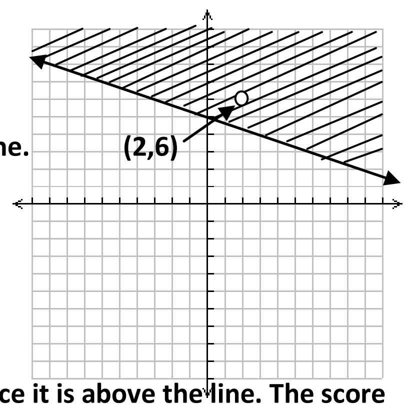
b. Graph the inequality.

Solution **a. Let the number of two-point score be x, and the number of three-point scores be y. Then the number of two-point scores times 2 and the number of three point scores times three have to be equal or exceed 15. The equation for this inequality can be written as follows:**

$$2x+3y\geq 15$$

b. In order to plot the equation you can solve the equation for y which yields:

$$y\geq -\frac{2}{3}x+5$$



The solution is the area on and above the line. That means any combination of integers for x and y with a coordinate point on or above the line will be a solution.

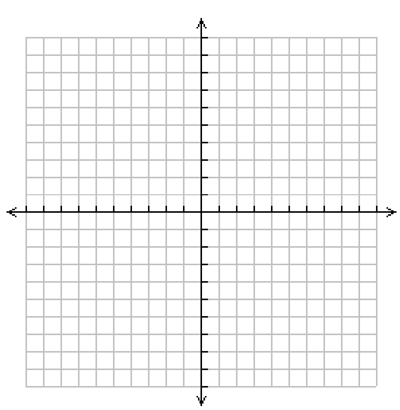
For example the point (2, 6) is a solution since it is above the line. The score associated with the point is 2x3 plus 6x2 which is larger or equal to 15.

PROBLEMS:

1. The Boston Celtic coach believes they can win the NBA title if they score at least 90 points per game.

a. Write an inequality to represent the number of two-point shots and the number of three-point shots that total at least 90 points per game.

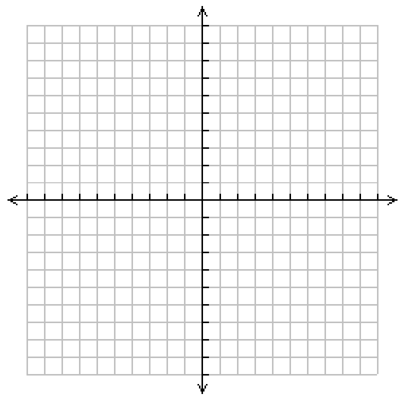
b. Graph the inequality.



2. In soccer you receive 3 points for winning and 1 point for a tie. In order to win the national championship a soccer coach estimates his team needs to make at least 50 points.

a. Write an inequality to represent the number of wins and ties that total at least 50 points for the season.

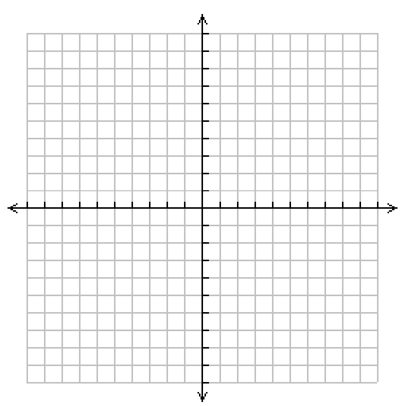
b. Graph the inequality.



3. You are the owner of a coffee shop and sell espressos for $2 and lattes for $4.

a. Write an inequality to represent the number of espressos and lattes you have to sell each day to make a daily revenue of more than $100.

b. Graph the inequality.



Lesson 2.6 Graph Systems of Linear Inequalities

Vocabulary

An example of a **system of linear inequalities** is: $x+2y<6$ and $3x-4y\geq 1$. A **solution of a system of inequalities** is an ordered pair that satisfies both inequalities.

Example 1 **Graph a system of two inequalities.**

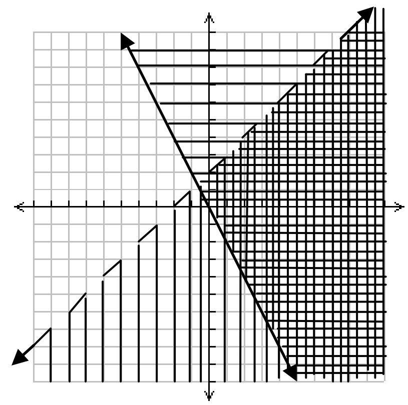
$$y < x + 2 \qquad Inequality\,1$$
$$y \geq -2x \qquad Inequality\,2$$

Solution

Step 1: Graph each inequality.

Step2: Identify the region that is common to both graphs.

The region with horizontal and vertical lines is the solution.

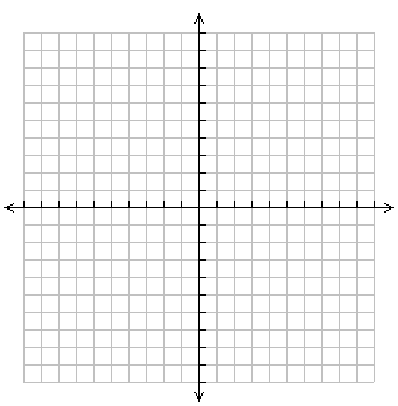


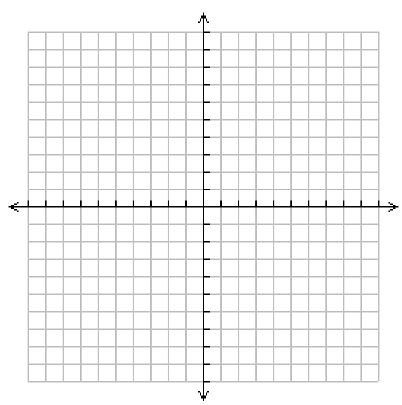
PROBLEMS: **Graph a system of two inequalities.**

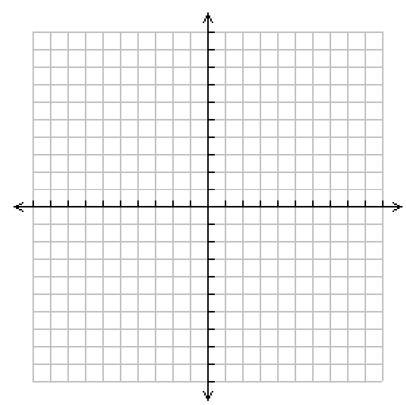
1. $y < x$
$y \geq 2$

2. $y < 2x + 1$
$y > -4x - 3$

3. $y < 2x$
$y \geq -3x$

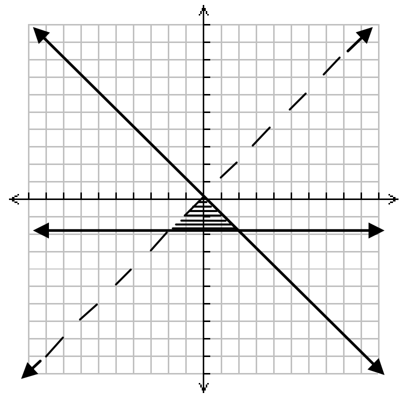






Example 2 **Graph a system of three or more inequalities.**

$$\begin{aligned} &y < x \quad \textit{Inequality } 1 \\ &y \geq -2 \quad \textit{Inequality } 2 \\ &y \leq -x \quad \textit{Inequality } 3 \end{aligned}$$



Solution

Step 1: Graph each inequality.

Step2: Identify the region that is common to all graphs.

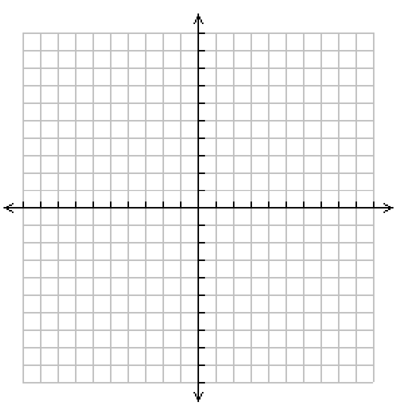
The region with horizontal and vertical lines is the solution.

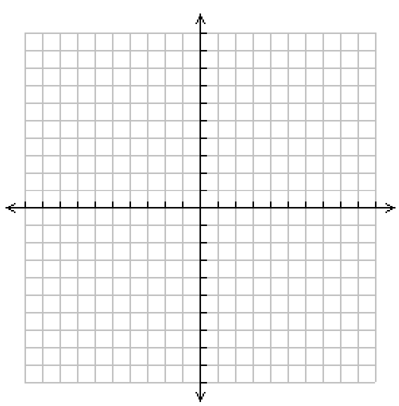
PROBLEMS: **Graph a system of inequalities.**

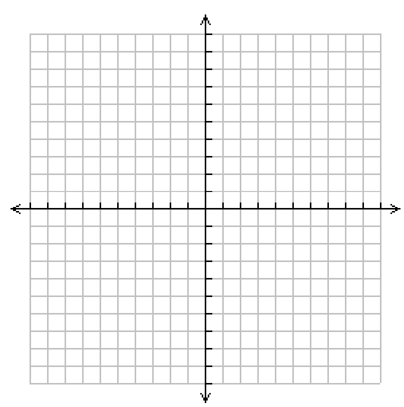
1. $y < x$, $y \geq 2$, $y < 5$

2. $y < x + 1$, $y > -3$, $x > 2$

3. $y \leq 3$, $y \geq 0$, $x \leq 2$, $x \geq -5$







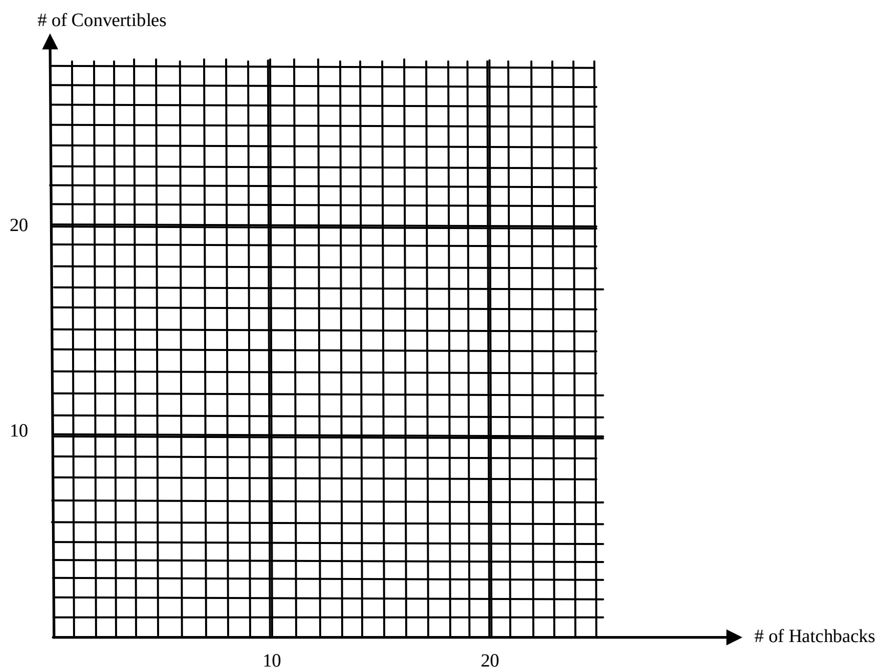
3. A car company produces 2 different cars at a plant: A hatchback and a convertible. The maximum production capacity is 20 automobiles per day. Each hatchback generates a profit of $1000 and each convertible generates a profit of $2000. The company wants to make at least $10,000 per day.

a. Write an inequality to represent the maximum production capacity for the plant.

b. Write an inequality to represent the least amount of profit desired per day.

c. Write a system of inequalities to represent both inequalities above (basically copy both equations).

d. Graph the system of inequalities and shade in the solutions.



e. The solution of the system is where both areas overlap. What does the area of overlap represent for this example?

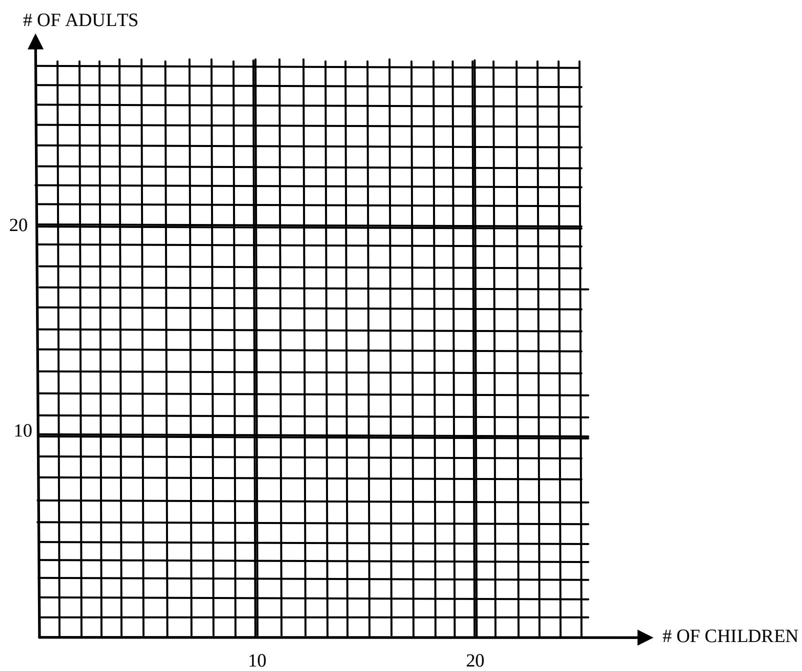
4. A small bus can carry a maximum load of 1600 pounds. A child (including luggage) weighs 100 pounds on average and an adult (including luggage) weighs 200 pounds on average. The child's fare is $1 and the adults fare is $5. The bus driver wants to make at least $60 per trip.

a. Write an inequality to represent the most weight the bus driver can have on the bus.

b. Write an inequality to represent the least amount of money for a trip.

c. Write a system of inequalities to represent both inequalities above (basically copy both equations).

d. Graph the system of inequalities and shade in the solutions.



e. The solution of the system is where both areas overlap. What does the area of overlap represent for this example?

5. You are shopping for a bike. The regular price of a bike is between \$200 and \$400. The store is running a sale where all bikes are between 20% and 60% off the regular price. The system of linear equations represents the amount you can save.

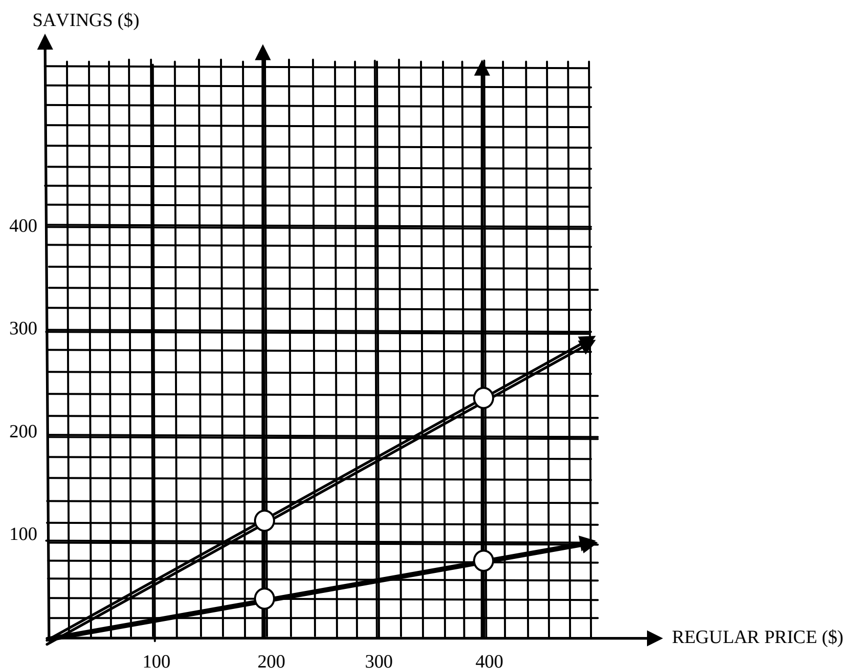
Let x represent the regular price.

Let y represent the amount you can save.

$x \geq 200$
$x \leq 400$ The regular price is between \$200 and \$400

$y \leq 0.60x$
$y \geq 0.20x$ The sales price is between 20% (=0.20) and 60% (=0.60) of the regular price

a. Plot the inequality



b. What is the most you can save? (Hint: Look at the point (400,240))

c. What is the least you can save? (Hint: Look at the point (200,40))

d. What is the most you will pay for the most expensive bike? (Hint: Look at the point (400,80))

e. What is the most that you will pay for the least expensive bike? (Hint: Look at the point (200,40))

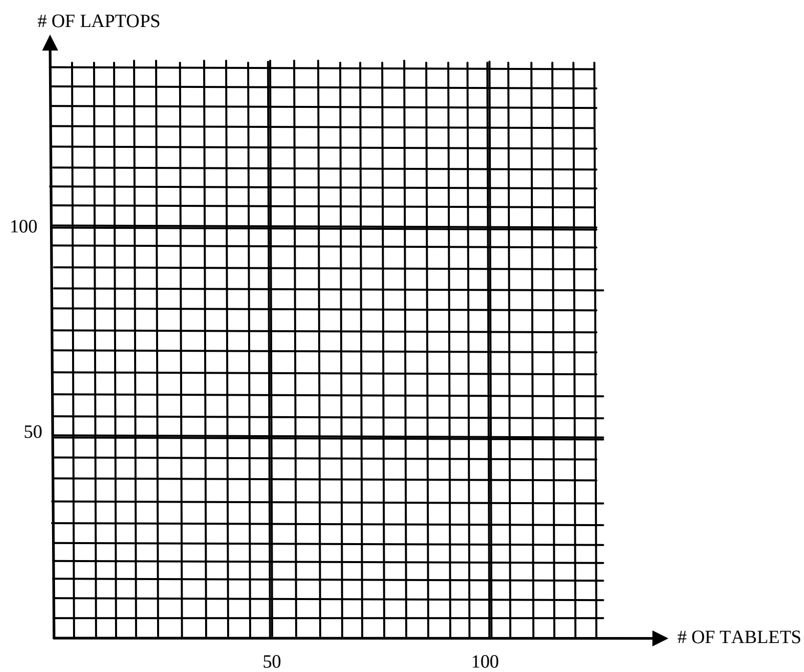
f. What is the least you will pay for the most expensive bike? (Hint: Look at the point (200,120))

6. A company is manufacturing computers. A tablet computer costs $300 to make and a laptop costs $600 to make. The budget available for materials is $21,000 per day. The manufacturing capacity is 50 computers per day.

Let x represent the tablets.
Let y represent the laptops.

a. Write a system of linear inequalities to represent the problem situation.

b. Graph the inequality.



Lesson 2.7 Graph Systems of Linear Inequalities Using Technology

You can use a graphing calculator (TI-83) to graph a system of linear inequalities.

STEP 1: Press **Y=** and enter the two inequalities as $\mathbf{Y_1}$ and $\mathbf{Y_2}$.

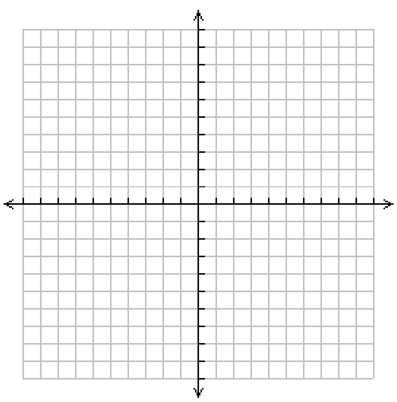
STEP 2: While in the **Y**= mode, access the inequality function by moving the cursor to the left until the **“\”** flashes. Press **ENTER** to select the appropriate inequality:
Flashing triangle on right top: **Y>**
Flashing triangle on left bottom: **Y<**

STEP 3: Press **Window** to set the bounds.

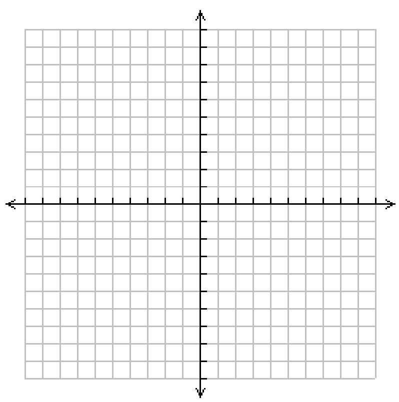
Step 4: Press **Graph.**

PROBLEMS: **Graph a system of inequalities using a graphing calculator.**

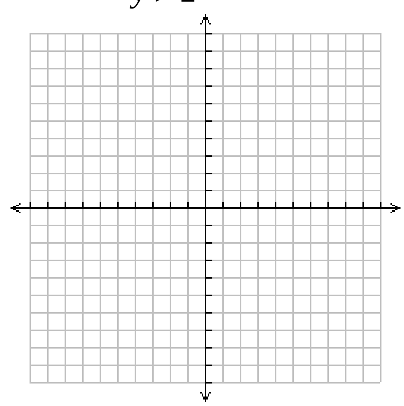
1. $y < 2x + 3$
$y \geq -4x$



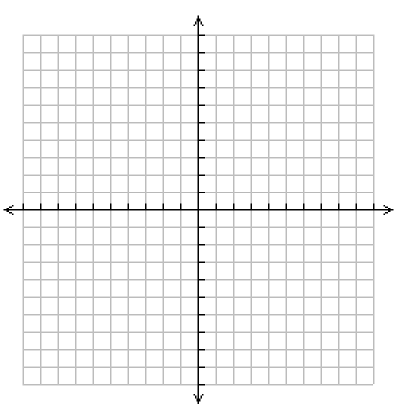
2. $y < x + 1$
$y > x - 2$



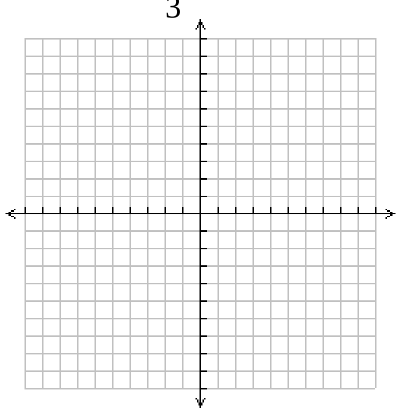
3. $y \leq \frac{1}{2}x - 3$
$y > 2$



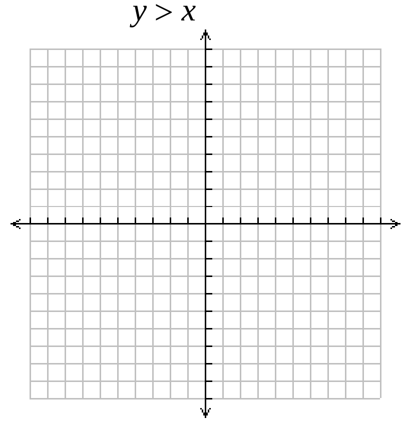
4. $y < x + 3$
$y < 5$



5. $y < 2x$
$y > \frac{1}{3}x + 5$



6. $y \leq -\frac{5}{6}x - 3$
$y > x$



Lesson 2.8 EOCT Problems

1. A manager is comparing the cost of buying ball caps with the company emblem from two different companies.

- Company X charges a \$50 fee plus \$7 per cap.
- Company Y charges a \$30 fee plus \$9 per cap.

For what number of ball caps will the manager's cost be the same for both companies?

A. 10 caps

B. 20 caps

C. 40 caps

D. 100 caps

2. A shop sells one-pound bags of peanuts for \$2 and three-pound bags of peanuts for \$5. If 9 bags are purchased for a total cost of \$36, how many three-pound bags were purchased?

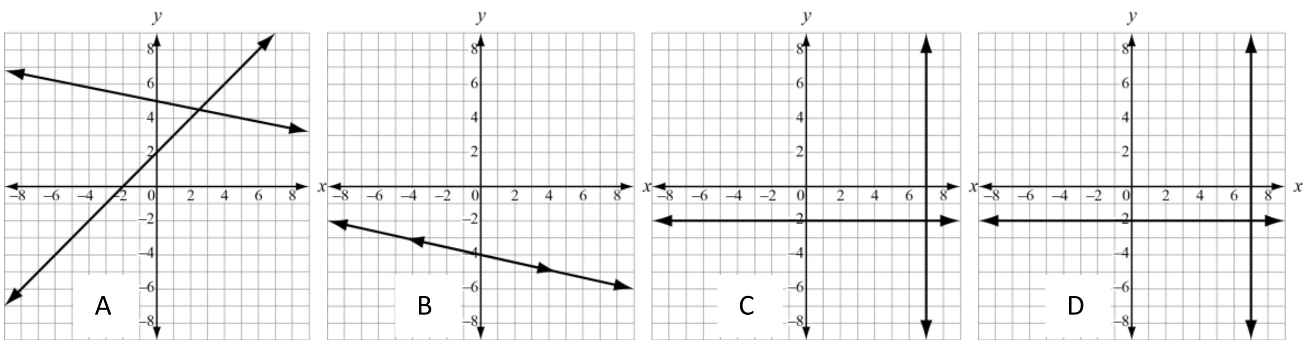
A. 3

B. 6

C. 9

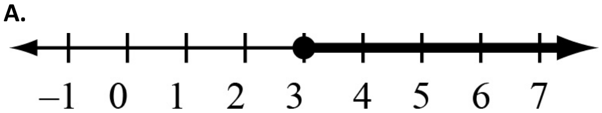
D. 18

3. Which graph would represent a system of linear equations that has multiple common coordinate pairs?

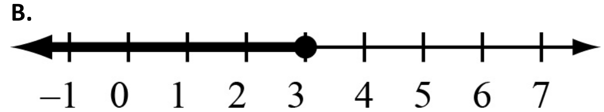


4. Which graph represents the solution to x > 3?

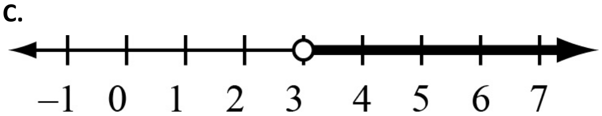
A.



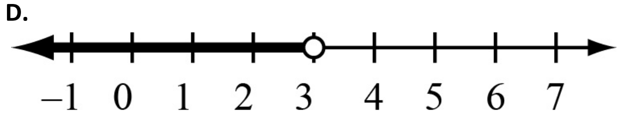
B.



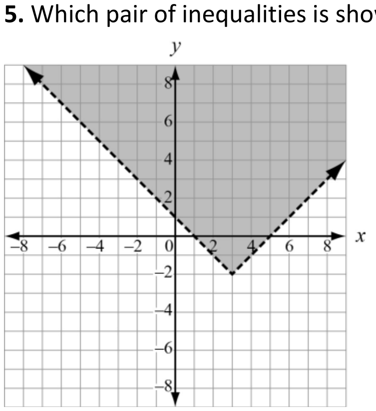
C.



D.



5. Which pair of inequalities is shown in the graph?



A. $y > -x + 1$ and $y > x - 5$

B. $y > x + 1$ and $y > x - 5$

C. $y > -x + 1$ and $y > -x - 5$

D. $y > x + 1$ and $y > -x - 5$